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COMPOSITE DESIGN SYNTHESIS

BATTELLE COLUMBUS LABORATORIES

PREPARED FOR
OFFICE OF NAVAL RESEARCH
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The research carried out deals with the problem of design synthesis in heterogeneous elasticity. Design synthesis is defined as the achievement of a desired design criterion, i.e., stress distribution, strength-to-weight ratio, etc., by preselecting a stress or displacement pattern in a stretched plate and then determining the variation of the elastic moduli that is required to permit the desired effects. This accomplishment requires the solution of the governing equations of

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elasticity, particularly the compatibility equation, in terms of preselected stress fields in the body of the plate for unknown material properties which are spatial functions.

During this work, solutions to the moduli variation problem for annular disks have been achieved for two stress criteria: constant hoop stress, and constant in-plane shear stress. The disk may be rotating and have boundary traction. A computer program, DOMOV1, was developed to carry out these solutions. Attempts to solve the moduli variation problem for a hole in an infinite plate (two-dimensional), subject to certain stress distributions, were unsuccessful. However, great insight was gained into this problem for future work.

Basically, this work has shown that the concept of design synthesis, as defined here, is a workable discipline and, in the case of rotating annular disks and pressurized thick-wall cylinders, can be applied utilizing the present state-of-the-art fabrication technology, but that its application to complex problems requires additional work.

FINAL TECHNICAL REPORT

on

COMPOSITE DESIGN SYNTHESIS

to

OFFICE OF NAVAL RESEARCH
DEPARTMENT OF THE NAVY

June 14, 1974

by

Milton Vagins

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June 14, 1974

Director
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Attention Program Management
DODAAD Code HX 1241

Dear Sir:

Form Approved Budget Bureau No. 22-R0293
Contract No. N00014-72-C0195

Enclosed is the Final Technical Report on the contract titled "Composite Design Synthesis".

Sincerely,

Milton Vagins
Principal Investigator
Applied Solid Mechanics Section

MV:jlg

Enclosures (3)

Final Technical Report

on

Composite Design Synthesis

14 June 1972

ARPA Order Number

1977

Program Code Number

2N10

Contract Number

N00014-72-C-0195

Scientific Officer

Director
Structural Mechanics, Mathematical
and Information Science Division

Office of Naval Research
Department of the Navy

Name of Contractor

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30 December 1971

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REPORT SUMMARY

This is the Final Technical Report describing the work done and the accomplishments of the research effort for ONR-ARPA titled "Composite Design Synthesis".

The research carried out deals with the problem of design synthesis in heterogeneous elasticity. Design synthesis is defined as the achievement of a desired design criterion, i.e., stress distribution, strength-to-weight ratio, etc., by preselecting a stress or displacement pattern in a stretched plate and then determining the variation of the elastic moduli that is required to permit the desired effects. This accomplishment requires the solution of the governing equations of elasticity, particularly the compatibility equation, in terms of preselected stress fields in the body of the plate for unknown material properties which are spatial functions.

During this work, solutions to the moduli variation problem for annular disks have been achieved for two stress criteria; constant hoop stress; and constant in-plane shear stress. The disk may be rotating and have boundary traction. A computer program, DOMOV1, was developed to carry out these solutions. Attempts to solve the moduli variation problem for a hole in an infinite plate (two dimensional), subject to certain stress distributions were unsuccessful. However, great insight was gained into this problem for future work.

Basically, this work has shown that the concept of design synthesis, as defined here, is a workable discipline and in the case of rotating annular disks and pressurized thick-wall cylinders, can be applied utilizing the present state-of-the-art fabrication technology, but that its application to complex problems requires additional work.

SYMBOLS

Symbol	Quantity
a,b	inside and outside radius
a_{ij}	material coefficients
c	dimensionless shear-modulus coefficient $\left(\equiv \frac{E_\theta}{2G}\right)$
A_o, B_o, C_o, C_1, C_2	constants of integration
e	orthotropic ratio $\left(\equiv \frac{E_\theta}{E_r}\right)$
E_i	modulus of elasticity corresponding to the subscripted direction ($i = r,\theta$)
f	$\sqrt{e}$
$f_1(\theta), f_2(\theta)$	functions of θ only
G	modulus of rigidity $\left(\equiv G_{r\theta} \equiv \frac{1}{a_{66}}\right)$
g	acceleration of gravity
k	radius ratio ($\equiv b/a$)
k_1	orthotropic ratio ($\equiv e$)
k_2	Poisson's ratio in tangential direction ($\equiv \nu_{\theta r}$)
m	$\cos \phi$
n	$\sin \phi$
P	internal pressure
q	external pressure
r	radius
T	temperature difference function
u,v	displacements
R,X,Y	body forces
U_o	specific strain energy
U	total strain energy

Symbol	Quantity
α	coefficient of linear thermal expansion
β	exponent ($\equiv -k_1/k_2$)
γ	material density
ϵ	strain component with one or two subscripts
Θ	body force in tangential direction
θ	angular position
λ	exponent
ν	Poisson's ratio in tangential direction ($\equiv \nu_{\theta r}$)
ξ	exponent $\left(\equiv - \left[\frac{k_1 - k_2}{(1-k_2)k_2}\right]\right)$
ρ	dimensionless position ratio ($\equiv r/b$)
ϕ	coordinate system offset angle
Ψ	stress function
ω	rotational velocity
δ	variation factor
η	arbitrary function

INTRODUCTION

It has long been recognized that structural elements composed of composite materials, such as glass, boron, carbon, or other filaments, embedded in a suitable matrix, such as epoxy or polyester, offer outstanding strength-to-weight ratios. The potential of such materials is considered so great that material scientists and engineers believe that they will form the bulk of the structural materials of the future. Though the application of such materials has been a growing part of the state of the art for structural components, particularly in the aerospace industry, the translation of the concept of fibrous composites into a primary load carrying structure has been and remains a challenging process.

The present and growing use of structural elements fabricated from composite materials creates the need for the development of a rational analytic design basis, which to a great extent is presently non-existent. This is not to be construed as meaning that little or no research on composite materials has been carried out. On the contrary, a large amount of literature has been generated dealing with both the determination of the mechanical properties of these materials and the analysis of specific structures fabricated from them.

In general, from the microscopic viewpoint, research on the mechanical properties of composites has dealt with the determination of such properties for materials having given component elements ordered in fixed spatial relationships. The spatial relation in these cases might have a high degree of symmetry, as in long or continuous filament composites, or a completely random or homogeneously disordered array as usually employed in short carbon or boron fiber composites. Such research has been directed towards the creation of analytic or experimental methods of determining the mechanical properties of composite structures in terms of the known properties of the composites' components, characteristic of this approach are the works of Sayers and Hanley [1]*, Chen and Cheng [2], Hill [3], and Gaonkar [4], among others.

In the analysis of structures composed of such composites, the material has generally been treated as exhibiting gross, homogeneous,

---

*Numbers in brackets are references found at the end of this report.

isotropic or anisotropic mechanical properties. These gross properties, when entered into the constitutive equations defining the material, have allowed analyses of such structures through the classical methods of the theory of elasticity, plates and shells, vibration, and others. Along these lines, the concept of the "unidirectional lamina" was introduced and utilized as the "fundamental unit of material" in design and analysis. This procedure employs test data obtained from a unidirectional lamina as the basis for the design of laminated components and structures. Exemplary work done along these lines has been carried out by Dong [5], Tsai [6,7], Tsai and Azzi [8], Whitney [9], and Whitney and Leissa [10,11], also among others.

All of these analyses have dealt with materials and structures that have predetermined mechanical and behavioral characteristics. That is, once the geometric array and the constituents of the composite are prescribed, then the mechanical properties of the material and the response characteristics of a structure fabricated from such a material have been inherently established. Analysis will merely determine what these properties and response characteristics are.

When dealing with composite materials, the analytical procedures discussed above appear to be highly inefficient in many applications. The designer of fibrous composite structures is presented with numerous degrees of freedom and an opportunity to exercise ingenuity totally unavailable to him with conventional materials. Composites, whether filamentary, fibrous, or sintered or fused metallics, are capable of being tailored to meet specific requirements. When considering specific structural applications for such materials, it would be logical to assume that a structure could be optimized, depending, of course, upon the optimization criterion, by varying the mechanical properties of the material throughout the structure. Further it would be logical to bypass analysis completely and define this now nonhomogeneous structure by some means of design synthesis. Admittedly, the creation of a design synthesis procedure to adequately handle most problems in structural design is quite difficult. However, the concept of design synthesis to determine the variation of the mechanical properties of a material within a structure so as to achieve a desired stress or deformation pattern in that structure is one capable of being developed.

The work described herein deals with the first phase of an effort to develop a design synthesis methodology for composites. It is directed specifically to the problem of heterogeneous plane elasticity.

GENERAL DISCUSSION

Consider the following question:

Given a plane elastic body with known boundary tractions and/or displacements, can the mechanical properties of the continuum be described such that an "arbitrary" stress distribution within the body is met?

The term "arbitrary" is to be understood as defining a family of stress distributions that are preselected but still conform to equilibrium requirements and boundary conditions. To answer this question we start by making the following two basic assumptions:

(1) the classical equations of linear elasticity are valid in this application, and

(2) the mechanical properties of the continuum can be expressed as spatial functions.

Following from these assumptions the well known governing relations of generalized plane stress, given in rectangular coordinates are as follows:

Equilibrium Equations

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + X = 0$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + Y = 0 , \qquad (1)$$

Strain-Displacement Equations

$$\epsilon_x = \frac{\partial u}{\partial x} , \quad \epsilon_y = \frac{\partial v}{\partial y} ,$$

$$\epsilon_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} , \qquad (2)$$

Compatibility Equation

$$\frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} = \frac{\partial^2 \epsilon_{xy}}{\partial x \partial y} . \qquad (3)$$

Assuming that the continuum exhibits orthotropic material properties and neglecting time and strain rate effects, the constitutive equations can be expressed as generalized Hooke's Law as

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 0 & a_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} + \begin{Bmatrix} \alpha_1 T \\ \alpha_2 T \\ 0 \end{Bmatrix} \tag{4}$$

where α_1 and α_2 are the coefficients of thermal expansion and T represents the temperature difference distribution. The relations defined by Equations (4) are valid when the axes of the material properties of the continuum are coincident with the axes selected for the differential equations of the problem. If the axes are not coincident, except for the z-axes, and the other two axes of the material properties are rotated about the z-axis through some angle, ϕ, in relation to the geometric axes, then more complicated relations between the stresses, temperature and strains are developed. Lekhnitskii [12] presents these relationships in some detail. For the generalized plane stress case in point the material coefficients are related to the two axis system by

$$\begin{bmatrix} a'_{11} \\ a'_{12} \\ a'_{21} \\ a'_{22} \\ a'_{66} \end{bmatrix} = \begin{bmatrix} m^4 & m^2n^2 & m^2n^2 & n^4 & 4m^2n^2 \\ m^2n^2 & m^4 & n^4 & m^2n^2 & -4m^2n^2 \\ m^2n^2 & n^4 & m^4 & m^2n^2 & -4m^2n^2 \\ n^4 & m^2n^2 & m^2n^2 & m^4 & 4m^2n^2 \\ m^2n^2 & -m^2n^2 & -m^2n^2 & m^2n^2 & (m^2-n^2) \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{21} \\ a_{22} \\ a_{66} \end{bmatrix} \tag{5}$$

where $m = \cos\phi$, and $n = \sin\phi$.

No generality will be lost by continuing with the constitutive equation as given by Equations (4). Substituting Equations (4) into (3),

assuming that the a_{ij}'s are spatially dependent and carrying out the required differentiation yields:

$$\left\{a_{12}\frac{\partial^2\sigma_y}{\partial y^2} + a_{66}\frac{\partial^2\sigma_{xy}}{\partial x\partial y} + a_{11}\frac{\partial^2\sigma_x}{\partial y^2} + a_{21}\frac{\partial^2\sigma_x}{\partial x^2} + a_{22}\frac{\partial^2\sigma_y}{\partial x^2}\right.$$

$$\left. + \frac{\partial^2}{\partial y^2}(\alpha_1 T) + \frac{\partial^2}{\partial x^2}(\alpha_2 T)\right\}$$

$$+\left\{\frac{\partial^2 a_{11}}{\partial y^2}\sigma_x + \frac{\partial^2 a_{12}}{\partial y^2}\sigma_y + \frac{\partial^2 a_{21}}{\partial x^2}\sigma_x + \frac{\partial^2 a_{22}}{\partial x^2}\sigma_y\right.$$

$$+\frac{\partial^2 a_{66}}{\partial x\partial y}\sigma_{xy} + 2\frac{\partial^2 a_{11}}{\partial y}\cdot\frac{\partial\sigma_x}{\partial y} + 2\frac{\partial a_{12}}{\partial y}\cdot\frac{\partial\sigma_y}{\partial y} + 2\frac{\partial a_{21}}{\partial x}\cdot\frac{\partial\sigma_x}{\partial x}$$

$$\left.+\frac{\partial a_{22}}{\partial x}\cdot\frac{\partial\sigma_y}{\partial x} + \frac{\partial a_{66}}{\partial x}\cdot\frac{\partial\sigma_{xy}}{\partial y} + \frac{\partial a_{66}}{\partial y}\cdot\frac{\partial\sigma_{xy}}{\partial x}\right\} = 0\,. \tag{6}$$

Assume that the body forces have a potential, V, such that

$$X = -\frac{\partial V}{\partial x}$$

$$Y = -\frac{\partial V}{\partial y}\,, \tag{7}$$

and choose Airy's stress function, Ψ, in the form

$$\sigma_x = \frac{\partial^2\Psi}{\partial y^2} + V$$

$$\sigma_y = \frac{\partial^2\Psi}{\partial x^2} + V \tag{8}$$

$$\sigma_{xy} = -\frac{\partial^2\Psi}{\partial x\partial y}\,.$$

Making the appropriate substitutions into Equation (6) yields

$$\left\{a_{22}\frac{\partial^4\psi}{\partial x^4} + (a_{21} + a_{12} - a_{66})\frac{\partial^4\psi}{\partial x^2\partial y^2} + a_{11}\frac{\partial^4\psi}{\partial y^4}\right.$$

$$\left. + (a_{11} + a_{12})\frac{\partial^2 V}{\partial y^2} + (a_{22} + a_{21})\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2}{\partial y^2}(\alpha_1 T) + \frac{\partial^2}{\partial x^2}(\alpha_2 T)\right\}$$

$$+ \left\{\left(\frac{\partial^2 a_{11}}{\partial y^2} + \frac{\partial^2 a_{21}}{\partial x^2}\right)\cdot\frac{\partial^2\psi}{\partial y^2} + \left(\frac{\partial^2 a_{12}}{\partial y^2} + \frac{\partial^2 a_{22}}{\partial x^2}\right)\cdot\frac{\partial^2\psi}{\partial x^2}\right.$$

$$- \frac{\partial^2 a_{66}}{\partial x\partial y}\cdot\frac{\partial^2\psi}{\partial x\partial y} + \frac{\partial^2}{\partial x^2}(a_{22} + a_{21})\cdot V + \frac{\partial^2}{\partial y^2}(a_{11} + a_{12})\cdot V$$

$$+ \frac{\partial}{\partial y}(2a_{12} - a_{66})\cdot\frac{\partial^3\psi}{\partial x^2\partial y} + \frac{\partial}{\partial x}(2a_{21} - a_{66})\cdot\frac{\partial^3\psi}{\partial x\partial y^2} + 2\frac{\partial a_{11}}{\partial y}\cdot\frac{\partial^3\psi}{\partial y^3}$$

$$\left. + 2\frac{\partial a_{22}}{\partial x}\cdot\frac{\partial^3\psi}{\partial x^3} + \frac{\partial}{\partial y}(2a_{11} + 2a_{12})\cdot\frac{\partial V}{\partial y} + \frac{\partial}{\partial x}(2a_{21} + 2a_{22})\cdot\frac{\partial V}{\partial x}\right\} = 0 \qquad (9)$$

and, of course, the equilibrium equations are met exactly.

Notice that in Equations (4), (5), and (9), a_{12} has not been equated to a_{21}. Normally, under the limit of small displacement theory dealing with linearly elastic materials which are conservative, the material coefficient matrix, defined in Equation (4), would be symmetric and a_{12} would equal a_{21}. However, some recent work by Bert and Guess [13], among others, shows that there exists experimentally derived data which indicate that for some types of composite materials exhibiting orthotropic properties the material coefficient matrix is not symmetric and a_{12} is not equal to a_{21}. To limit the growing complexity of this work, the material coefficient matrix will be taken as symmetric, at least for the initial phase of this effort.

Equation (9) can also be expressed in polar coordinates as follows:

(Note: Here, the a_{ij}'s are in polar coordinates and are not equal to the a_{ij}'s for rectangular coordinates.)

$$
\begin{aligned}
&\Bigg\{ \Bigg[a_{22} \frac{\partial^4 \psi}{\partial r^4} + 2 \frac{a_{22}}{r} \frac{\partial^3 \psi}{\partial r^3} - \frac{a_{11}}{r^2} \frac{\partial^2 \psi}{\partial r^2} + \frac{a_{11}}{r^3} \frac{\partial \psi}{\partial r} + \frac{a_{11}}{r^4} \frac{\partial^4 \psi}{\partial \theta^4} \\
&\quad - \left(\frac{2a_{12} + a_{66}}{r^3} \right) \cdot \frac{\partial^3 \psi}{\partial r \partial \theta^2} + \left(\frac{2a_{12} + a_{66}}{r^2} \right) \cdot \frac{\partial^4 \psi}{\partial r^2 \partial \theta^2} \\
&\quad + \left(\frac{2a_{11} + 2a_{12} + a_{66}}{r^4} \right) \frac{\partial^2 \psi}{\partial \theta^2} \Bigg] \\
&\quad + \left[a_{21} \frac{\partial^2}{\partial r^2} + \left(\frac{2a_{21} - a_{11}}{r} \right) \frac{\partial}{\partial r} + \frac{a_{11}}{r^2} \frac{\partial^2}{\partial \theta^2} \right] \left(V_r \right) \\
&\quad + \left[a_{22} \frac{\partial^2}{\partial r^2} + \left(\frac{2a_{22} - a_{12}}{r} \right) \frac{\partial}{\partial r} + \frac{a_{12}}{r^2} \frac{\partial^2}{\partial \theta^2} \right] \left(V_\theta \right) \\
&\quad + \left[\frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} \right] \left(\alpha_\theta T \right) + \left[\frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} - \frac{1}{r} \frac{\partial}{\partial r} \right] \left(\alpha_r T \right) \Bigg\} \\
&+ \Bigg\{ \frac{\partial a_{12}}{\partial r} \cdot \left[\frac{1}{r} \frac{\partial^2 \psi}{\partial r^2} + \frac{2}{r^2} \frac{\partial^3 \psi}{\partial r \partial \theta^2} - \frac{2}{r^3} \frac{\partial^2 \psi}{\partial \theta^2} + \frac{2 \partial V_r}{\partial r} + \frac{2V_r - V_\theta}{r} \right] \\
&+ \frac{\partial a_{12}}{\partial \theta} \left[\frac{2}{r^2} \frac{\partial^3 \psi}{\partial r^2 \partial \theta} + \frac{2}{r^3} \frac{\partial V_\theta}{\partial \theta} \right] + \frac{\partial a_{22}}{\partial r} \left[2 \frac{\partial^3 \psi}{\partial r^3} + \frac{2}{r} \frac{\partial^2 \psi}{\partial r^2} + 2 \frac{\partial V_\theta}{\partial r} + \frac{2V_\theta}{r} \right] \\
&- \frac{\partial a_{11}}{\partial r} \left[\frac{1}{r^3} \frac{\partial^2 \psi}{\partial \theta^2} + \frac{1}{r^2} \frac{\partial \psi}{\partial r} + \frac{Vr}{r} \right] + \frac{\partial a_{11}}{\partial \theta} \left[\frac{2}{r^4} \frac{\partial^3 \psi}{\partial \theta^3} + \frac{2}{r^3} \frac{\partial^2 \psi}{\partial r \partial \theta} + \frac{2}{r^2} \frac{\partial V_r}{\partial \theta} \right] \\
&- \frac{\partial a_{66}}{\partial r} \left[\frac{1}{r^3} \frac{\partial^2 \psi}{\partial \theta^2} - \frac{1}{r^2} \frac{\partial^3 \psi}{\partial r \partial \theta^2} \right] + \frac{\partial a_{66}}{\partial \theta} \left[\frac{1}{r^4} \frac{\partial \psi}{\partial \theta} - \frac{1}{r^3} \frac{\partial^2 \psi}{\partial r \partial \theta} + \frac{1}{r^2} \frac{\partial^3 \psi}{\partial r^2 \partial \theta} \right] \\
&+ \frac{\partial^2 a_{12}}{\partial r^2} \left[\frac{1}{r^2} \frac{\partial^2 \psi}{\partial \theta^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + V_r \right] + \frac{\partial^2 a_{12}}{\partial \theta^2} \left[\frac{1}{r^2} \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} V_\theta \right] \\
&+ \frac{\partial^2 a_{22}}{\partial r^2} \left[\frac{\partial^2 \psi}{\partial r^2} + V_\theta \right] + \frac{\partial^2 a_{11}}{\partial \theta^2} \left[\frac{1}{r^4} \frac{\partial^2 \psi}{\partial \theta^2} + \frac{1}{r^3} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} V_r \right] \\
&+ \frac{\partial^2 a_{66}}{\partial r \partial \theta} \left[\frac{1}{r^2} \frac{\partial^2 \psi}{\partial r \partial \theta} - \frac{1}{r^3} \frac{\partial \psi}{\partial \theta} \right] \Bigg\} = 0 \qquad (10)
\end{aligned}
$$

with Ψ being the stress function defined by

$$\sigma_r = \frac{1}{r}\frac{\partial\Psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\Psi}{\partial\theta^2} + V_r$$

$$\sigma_\theta = \frac{\partial^2\Psi}{\partial r^2} + V_\theta \tag{11}$$

$$\sigma_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\Psi}{\partial\theta}\right) .$$

Equations (11) satisfy the equilibrium equations formulated in polar coordinates for plane stress which are

$$\frac{\partial\sigma_r}{\partial r} + \frac{1}{r}\frac{\partial\sigma_{r\theta}}{\partial\theta} + \frac{\sigma_r - \sigma_\theta}{r} + R = 0$$

$$\frac{\partial\sigma_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial\sigma_\theta}{\partial\theta} + 2\frac{\sigma_{r\theta}}{r} + \Theta = 0 . \tag{12}$$

In order for Equations (11) to meet Equation (12) exactly the body functions V_r and V_θ must be defined as

$$V_\theta - \frac{\partial}{\partial r}(rV_r) = R$$

$$-\frac{\partial V_\theta}{\partial\theta} = \Theta , \tag{13}$$

thus putting a rather narrow interpretation on the body forces.

In the classical approach to the problem of orthotropic plane elasticity, the coefficients a_{ij} are either constants, as in the case of the homogeneous condition, or as in some rare cases, are special functions of position. In the first case all the terms within the second set of large braces, $\{\,\}$, in Equations (9) and (10) are zero leaving the remainder of these two equations in the form of the well-known, homogeneous, orthotropic,

compatibility equations in terms of the stress function Ψ. In the secord case all the partial derivatives of the a_{ij}'s which appear in these second sets of braces are capable of being evaluated, resulting in extremely complicated fourth order partial differential equations with variable coefficients. Proceeding along classical lines, these equations must be solved for Ψ which contains arbitrary constants of integration. These constants are then determined by evaluating Ψ in terms of the stresses on the boundaries.

Suppose it is assumed that the material coefficients, the a_{ij}'s, are unknown but that the stress function Ψ is a fully defined function of the spatial coordinates. That is, the stresses throughout the body as well as on the boundary are known. In such a case, Equations (9) and (10) reduce to second order partial differential equations with variable coefficients, in terms of the a_{ij}'s. The solution of these equations and the resultant determination of the magnitude and distribution of the material properties throughout the body is defined in this work as design synthesis.

It is believed that Equations (9) and (10) have not been previously published. Bert [14] derived an equation similar to (10) wherein he reduced the unknown material property coefficients from four to one. His formulation is as follows:

$$
\begin{aligned}
& S\Big[\frac{\partial^4\Psi}{\partial r^4} + \frac{2}{r}\frac{\partial^3\Psi}{\partial r^3} - \frac{e}{r^2}\frac{\partial^2\Psi}{\partial r^2} + \frac{e}{r^3}\frac{\partial\Psi}{\partial r} + \frac{2(c-\nu)}{r^2}\frac{\partial^4\Psi}{\partial r^2\partial\theta^2} \\
& \quad - \frac{2(c-\nu)}{r^3}\frac{\partial^3\Psi}{\partial r\partial\theta^2} + \frac{2(c-\nu+e)}{r^4}\frac{\partial^2\Psi}{\partial\theta^2} + \frac{e}{r^4}\frac{\partial^4\Psi}{\partial\theta^4}\Big] \\
& + \frac{dS}{dr}\Big[2\frac{\partial^3\Psi}{\partial r^3} + \frac{2-\nu}{r}\frac{\partial^2\Psi}{\partial r^2} - \frac{e}{r^2}\frac{\partial\Psi}{\partial r} + \frac{2(c-\nu)}{r^2}\frac{\partial^3\Psi}{\partial r\partial\theta^2} \\
& \quad - \frac{2(c-\nu)+e}{r^3}\frac{\partial^2\Psi}{\partial\theta^2}\Big] + \frac{d^2S}{dr^2}\Big[\frac{\partial^2\Psi}{\partial r^2} - \frac{\nu}{r}\frac{\partial\Psi}{\partial r} - \frac{\nu}{r^2}\frac{\partial^2\Psi}{\partial\theta^2}\Big] \\
& \quad - \Big[\nu\frac{\partial^2}{\partial r^2} - \frac{e-2\nu}{r}\frac{\partial}{\partial r} - \frac{e}{r^2}\frac{\partial^2}{\partial\theta^2}\Big]\left(SV_r\right) \\
& \quad + \Big[\frac{\partial^2}{\partial r^2} + \frac{2+\nu}{r}\frac{\partial}{\partial r} - \frac{\nu}{r^2}\frac{\partial^2}{\partial\theta^2}\Big]\left(SV_\theta\right) = 0 \ , \qquad (14)
\end{aligned}
$$

with the thermal terms omitted. Equation (14) is equal to Equation (10) with the following identities:

$$
\begin{aligned}
S &\equiv a_{22} \\
e &\equiv a_{11}/a_{22} \\
c &\equiv a_{66}/2a_{22} \\
\nu &\equiv -\, a_{12}/a_{22}
\end{aligned}
\qquad (15)
$$

with S being the dependent variable and e, c, and ν being fixed ratios. It is anticipated that most real composite materials will exhibit material property characteristics as defined by Equations (15).

The solution of equations such as (9) and (10) where there are 4 or more independent a_{ij}'s, even where these a_{ij}'s are assumed independent of temperature, is quite difficult but certainly not impossible. The simplifying assumption made in Equations (15) leading to the formulation of Equation (14) reduces the problem to only one unknown parameter.

APPLICATIONS

Rotationally Symmetric Problems

Example 1: The pressurized annular disk, internal pressure: Consider a pressurized annular disk as shown in Figure 1. In the case where the ring is isotropic and homogeneous, the stress distribution is as developed by Lamé (1852) and is given by (c.f., Ref. [15], p. 60),

$$\sigma_r = -\frac{P}{k^2 - 1}\left(\frac{1}{\rho^2} - 1\right)$$

$$\sigma_\theta = -\frac{P}{k^2 - 1}\left(\frac{1}{\rho^2} + 1\right) \tag{16}$$

where $\rho = r/b$, and $k = b/a$. These relations lead to the following conclusions:

(1) $|\sigma_\theta| > |\sigma_r|$ for all ρ and all k

(2) $(\sigma_\theta)_{max}$ occurs at the inner boundary ($\rho = 1/k$), and thus

$$(\sigma_\theta)_{max} = P\left(\frac{k^2 + 1}{k^2 - 1}\right) > P \tag{17}$$

From the stresses so generated, which for the homogeneous ring, are quite independent of the material properties, it is clear that the material is not being used efficiently, particularly as the thickness ratio, k, increases.

For the homogeneous, orthotropic annular disk Bienick et al. [16] shows that the stresses are given by

$$\sigma_r = c_1 r^{-(f+1)} + c_2 r^{f-1}$$

$$\sigma_\theta = -c_1 f r^{-(f+1)} + c_2 f r^{f-1} \tag{18}$$

where $f = (a_{11}/a_{22})^{1/2}$, and c_1 and c_2 are determined from the boundary conditions, in this case $\sigma_r = -P$ at $r = a$ and $\sigma_r = 0$ at $r = b$. This work and

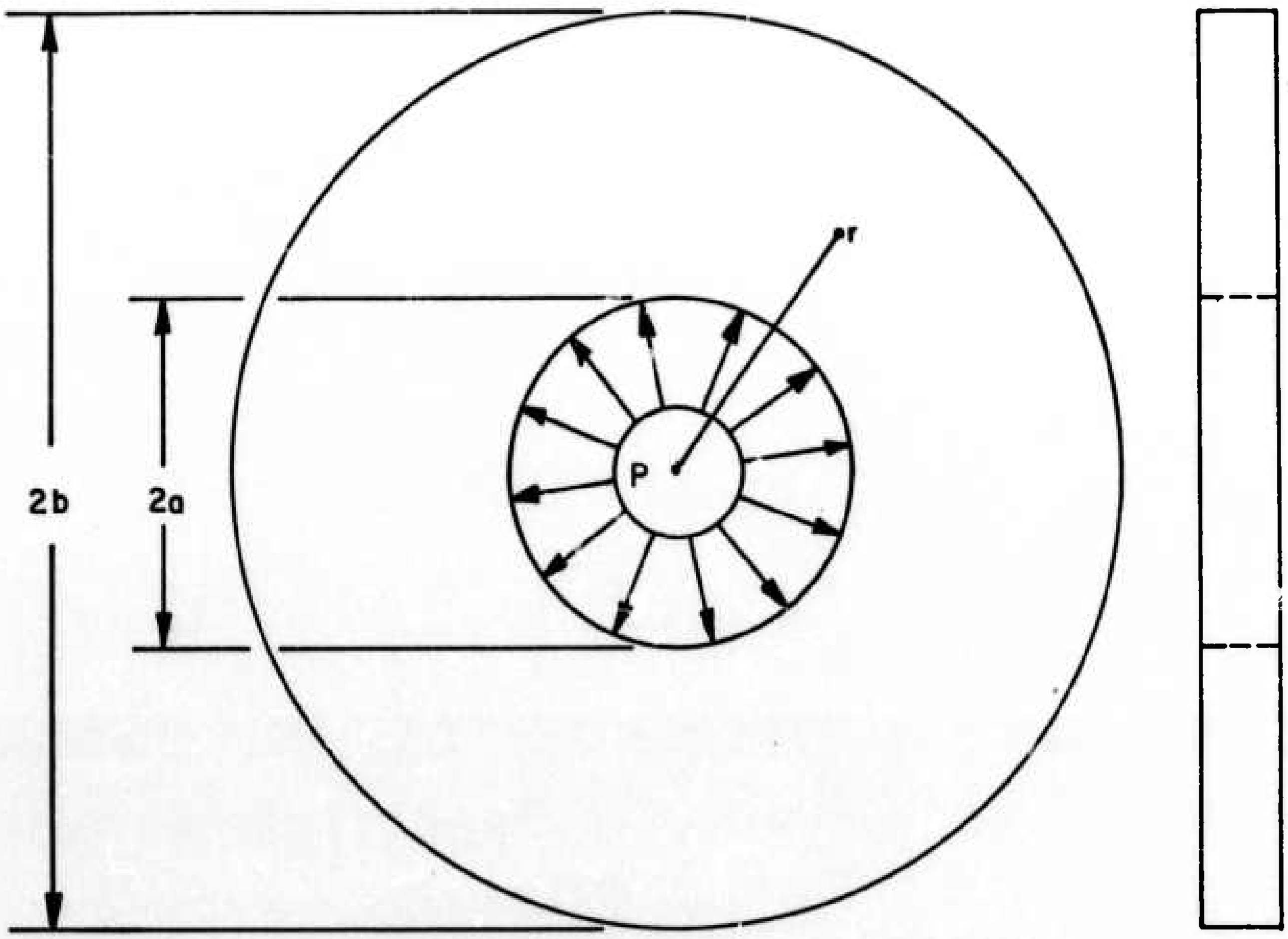

FIGURE 1. PRESSURIZED ANNULAR DISK

work done by Shaffer [17] show that as k gets large the limit of the stress concentration approaches the orthotropy ratio, f, and does so rapidly. Both Bienick and Shaffer deal with the nonhomogeneous case, but both assume a special form of nonhomogeneity, mainly that

$$a_{11} = \bar{a}_{11}\, r^{\lambda}$$

$$a_{22} = \bar{a}_{22}\, r^{\lambda}$$

$$a_{12} = a_{21} = \bar{a}_{12}\, r^{\lambda}$$

where λ is real and $\bar{a}_{11}$, $\bar{a}_{22}$, and $\bar{a}_{12}$ are constants. Both carry out optimization by varying λ and observing the results, but no attempt was made to carry out <u>design synthesis</u> directly.

Consider now that the disk is composed of a heterogeneous, orthotropic material where the material properties are undefined but are functions of the radius of the disk. The equilibrium equation for this case reduces to

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \; . \tag{19}$$

The compatibility equation reduces to

$$\frac{d}{dr}\,(r\epsilon_\theta) - \epsilon_r = 0 \; , \tag{20}$$

and the stress strain relation for an orthotropic medium is

$$\begin{aligned} \epsilon_r &= a_{11}\sigma_r + a_{12}\sigma_\theta \\ \epsilon_\theta &= a_{12}\sigma_r + a_{22}\sigma_\theta \end{aligned} \tag{21}$$

where a_{11}, a_{12}, and a_{22} are functions of r. Assuming a stress function such that

$$\sigma_r = \frac{1}{r}\Psi$$

$$\sigma_\theta = \frac{d\Psi}{dr} \tag{22}$$

and substituting Equations (21) and (22) into Equation (20) and carrying out the required differentiation results in

$$a_{22}\Psi'' + (a'_{22} + \frac{1}{r}a_{22})\Psi' + (\frac{1}{r}a'_{12} - \frac{1}{r^2}a_{11})\Psi = 0 , \tag{23}$$

where the prime marks designate absolute derivatives with respect to r.

Equation (23) is a single differential equation with three independent material coefficients as the operational parameters. When dealing with specific materials they may be found to be completely independent or show some type of defined relationship. As a first step we will assume that there does exist a simple ratio relationship between them that is expressable as

$$a_{22} = a_{22} \quad , \quad a'_{22} = a'_{22}$$

$$a_{11} = k_1 a_{22}, \quad a'_{11} = k_1 a'_{22} \tag{24}$$

$$a_{12} = k_2 a_{22}, \quad a'_{12} = k_2 a'_{22} .$$

Making the appropriate substitutions in Equation (23) yields

$$\left[\Psi' + \frac{k_2}{r}\Psi\right] a'_{22} + \left[\Psi'' + \frac{1}{r}\Psi' - \frac{k_1}{r^2}\Psi\right] a_{22} = 0 . \tag{25}$$

Thus if Ψ is known a_{22} is fully defined. Let us suppose that for effective material utilization it is desired that σ_θ be constant throughout the disk; i.e., $\sigma_\theta = A_o$. Integrating the second of Equations (22) gives

$$\Psi = A_o r + B_o . \tag{26}$$

Applying the boundary conditions that $\sigma_r(a) = - P$ and $\sigma_r(b) = 0$ yields

$$\Psi = \frac{Pb}{k - 1} [\rho - 1]$$

$$\sigma_r = - \frac{P}{k - 1} [\frac{1}{\rho} - 1] \qquad (27)$$

$$\sigma_\theta = \frac{P}{k - 1}$$

with $\rho = r/b$ and $k = b/a$ as before. It is interesting to compare Equations (27) with Equations (16) and (17). From Equations (27) it can be seen that if k is greater than 2, σ_θ becomes less than the pressure P, and σ_r, which equals P at $\rho = 1/k$, becomes the maximum normal stress in absolute value. Thus for such a stress function and geometry there is no effective stress concentration. This is, of course, never true for the homogeneous disk.

With the stress function now fully defined, Equation (25) becomes

$$\left[(1+k_2) - \frac{k_2 b}{r}\right] a'_{22} + \left[\frac{(1-k_1)}{r} + \frac{k_1 b}{r^2}\right] a_{22} = 0 . \qquad (28)$$

which has the solution

$$a_{22} = C_o(\rho)^{\beta-\xi} \left[\frac{k_2}{\rho} + (1-k_2)\right]^{-\xi} \qquad (29)$$

where $\rho = r/b$

$\beta = - k_1/k_2$

$\xi = - \left[\frac{k_1-k_2}{(1-k_2)k_2}\right]$,

and for the modified orthotropic condition as defined by Equations (21) and (24)

$$a_{22} = a_{22} = \frac{1}{E_\theta}$$

$$k_1 = \frac{a_{11}}{a_{22}} = \frac{E_\theta}{E_r} \qquad (30)$$

$$k_2 = -\frac{a_{12}}{a_{22}} = \nu_{\theta r} .$$

The inverse of Equation (29) or $E_\theta(r)$ is shown plotted in Figure 2 for various k_1's while k_2 was held constant at 0.5. This figure shows the strong dependency of the modulus distribution upon the orthotropy ratio k_1. The ratio k_2 has a lesser effect as shown by Figure 3. Here the isotropic case ($k_1 = 1$) is shown plotted for five values of k_2.

Example 2. Pressurized annular disk, external pressure: Consider the pressurized annular disk as shown in Figure 1 but with the pressure acting on the OD rather than the ID as shown. If the stress criterion is retained; i.e., σ_θ = constant, then the stress function becomes

$$\begin{aligned} \Psi &= \frac{qb}{k-1}\,[1 - \rho k] \\ \sigma_r &= \frac{q}{k-1}\,[\frac{1}{\rho} - k] \qquad (31) \\ \sigma_\theta &= -\,\frac{qk}{k-1} \end{aligned}$$

where q = external pressure

$k = b/a$

$\rho = r/b$.

The compatibility equation (25) becomes

$$\left[(1+k_2) - \frac{k_2 a}{r}\right] a'_{22} + \left[\frac{(1-k_1)}{r} + \frac{k_1 a}{r^2}\right] a_{22} = 0 \,. \qquad (32)$$

Equation (32) is the same as Equation (28) except that a replaces b. The solution of Equation (32) is

$$a_{22} = C_o(\rho)^{\beta-\xi}\left[\frac{k_2}{\rho k} + (1-k_2)\right]^{-\xi} \qquad (33)$$

where β, ξ, and ρ are defined in Equation (29). Comparing Equations (33) and (29) we see that they are not the same, differing by the quantity 1/k

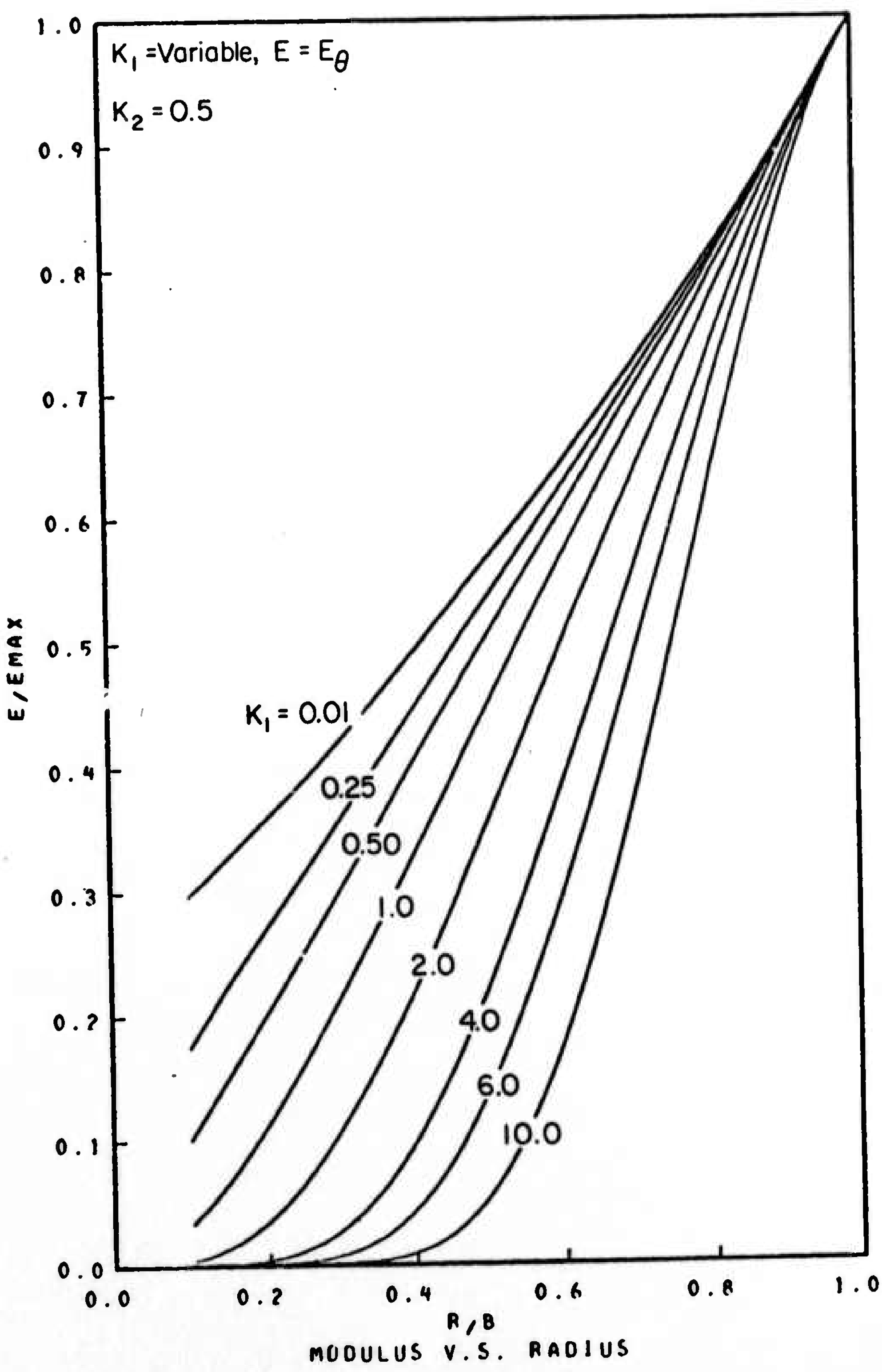

FIGURE 2. MODULUS VARIATION FOR ORTHOTROPIC DISK WITH PRESSURIZED I.D., WITH σ_θ = Constant

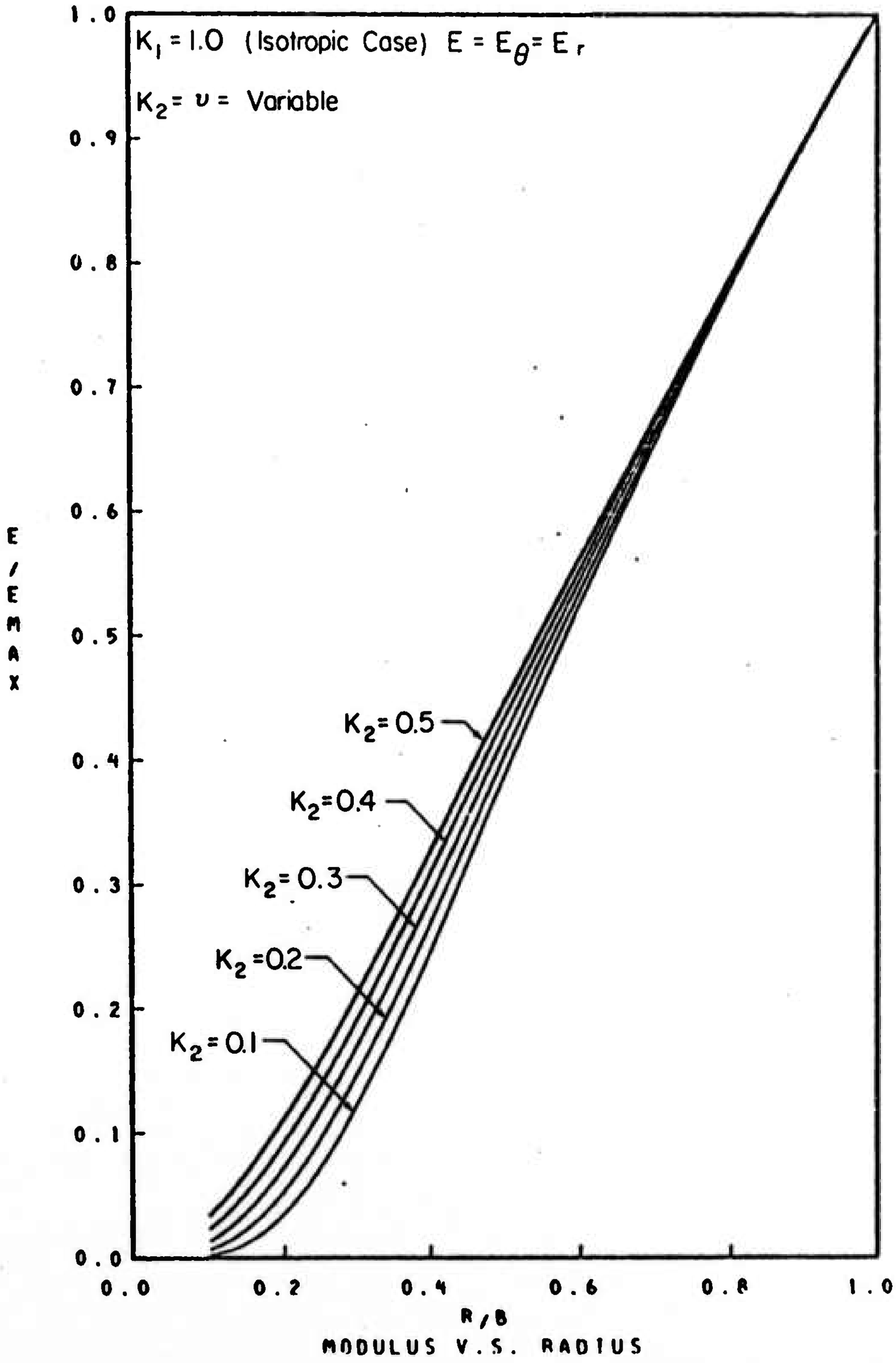

FIGURE 3. MODULUS VARIATION FOR ISOTROPIC DISK WITH PRESSURIZED I.D., WITH σ_θ = Constant

within the second factor. Thus the distribution of the material parameters through the thickness of the disk must be different as compared to the internally pressurized disk. This is shown in Figures 4 and 5. Also notice that for the internally pressurized disk the hoop stress σ_θ tends to zero as k tends to infinity while the limit on σ_θ for the externally pressurized disk is q, the pressure. Thus for the externally pressurized case the material is not being as effectively utilized and perhaps some other criterion might more suitably apply.

If the annulus has both internal and external pressure and the same stress criterion is applied, i.e., σ_θ = constant, then

$$\Psi = \frac{Pb}{k-1}\left[\rho - 1\right] - \frac{qb}{k-1}\left[1 - \rho k\right]$$

$$\sigma_r = -\frac{P}{k-1}\left[\frac{1}{\rho} - 1\right] - \frac{q}{k-1}\left[\frac{1}{\rho} - k\right] \tag{34}$$

$$\sigma_\theta = \frac{1}{k-1}\left[P - qk\right],$$

and the material property variation is given by

$$a_{22} = C_o(\rho)^{\beta-\xi}\left[\frac{k_2}{\rho} + (1-k_2)\left(\frac{Pa-qb}{Pa-qa}\right)\right]^{-\xi} \tag{35}$$

Example 3: The rotating disk: Consider the rotating, uniform thickness annular disk as shown in Figure 6. The equilibrium equation governing this case is

$$\frac{d}{dr}(r\sigma_r) - \sigma_\theta + \frac{\gamma}{g}\omega^2 r^2 = 0 . \tag{36}$$

The compatibility equation in terms of strain and the stress-strain relations for an orthotropic material are given by Equations (20) and (21), respectively. Substituting Equation (21) into (20) results in

$$0 = a'_{12}\sigma_r + a_{12}\sigma'_r + a'_{22}\sigma_\theta + a_{22}\sigma'_\theta + \frac{1}{r}\left[(a_{12}-a_{11})\sigma_r + (a_{22}-a_{12})\sigma_\theta\right]. \tag{37}$$

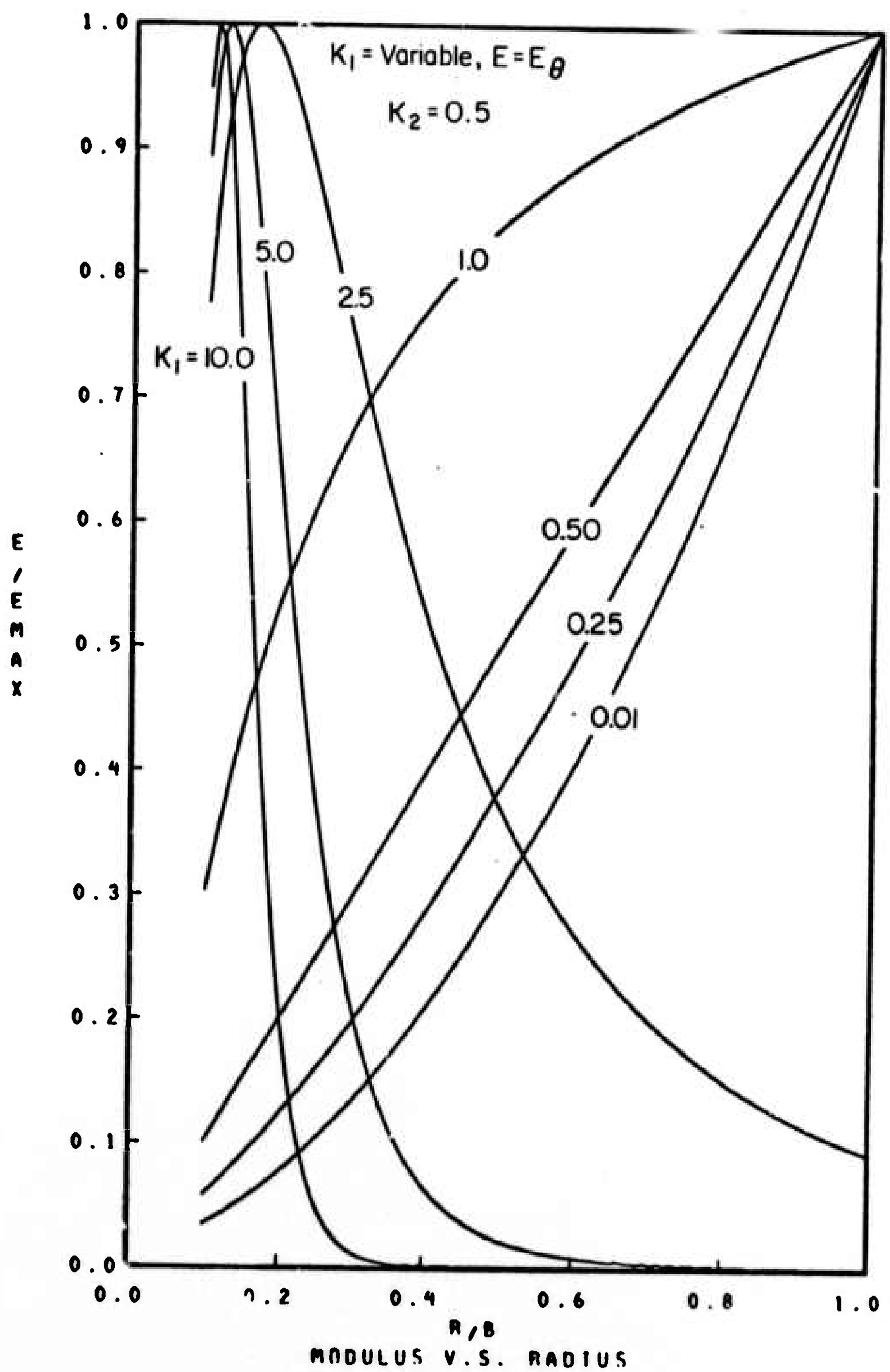

FIGURE 4. MODULUS VARIATION FOR ORTHOTROPIC DISK WITH PRESSURIZED O.D., WITH σ_θ = Constant

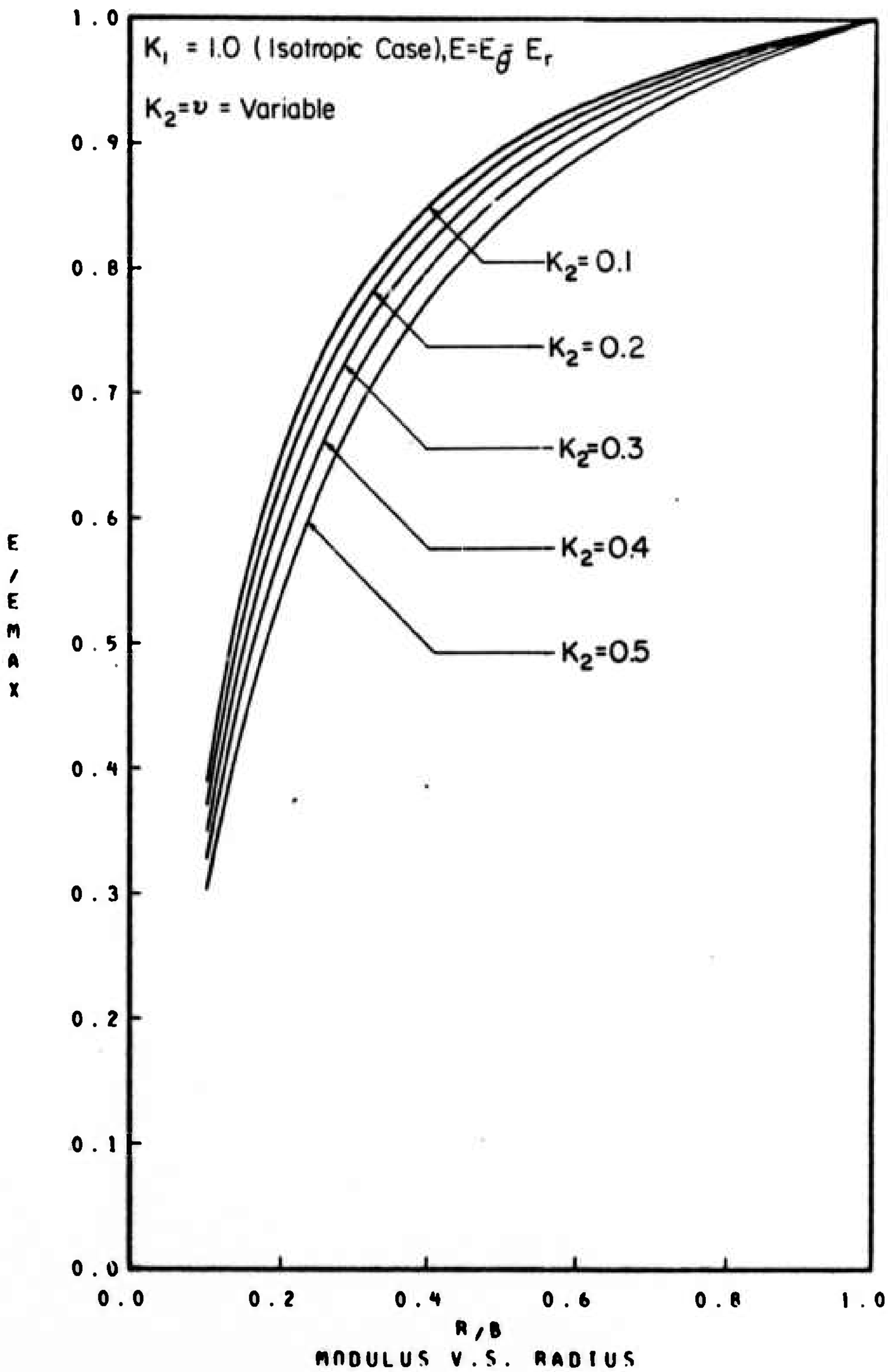

FIGURE 5. MODULUS VARIATION FOR ISOTROPIC DISK WITH PRESSURIZED O. D., WITH σ_θ = CONSTANT

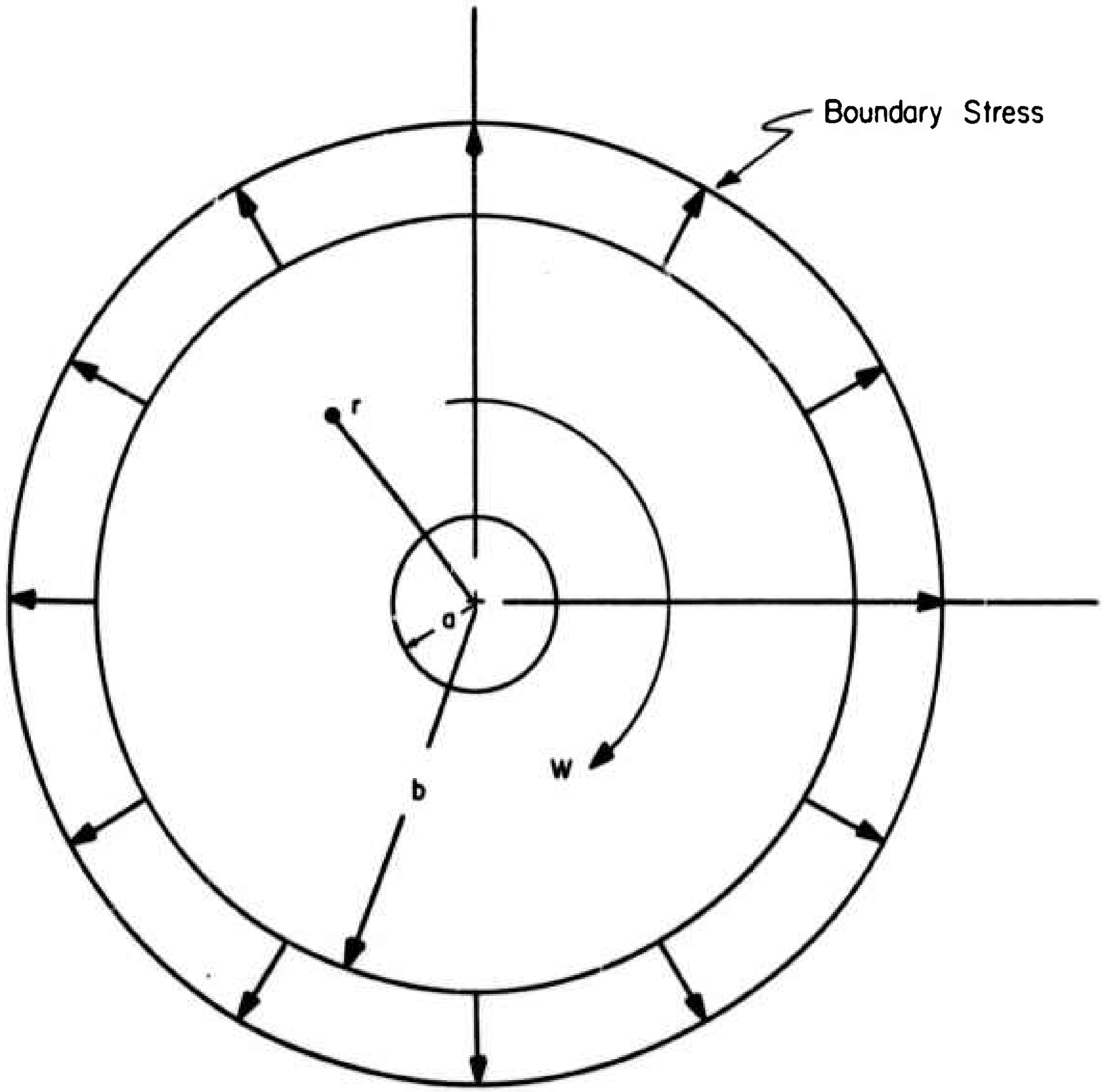

FIGURE 6. ROTATING ANNULAR DISK

Assume the stress function

$$\sigma_r = \frac{1}{r}\Psi$$

$$\sigma_\theta = \Psi' + \frac{\gamma}{g}\omega^2 r^2 \ . \qquad (38)$$

It is possible that the material density, γ, could be a function of the radius. At this stage such a condition introduces an unnecessary complication and will not be considered. This being the case the governing compatibility equation becomes

$$\Psi''(a_{22}) + \Psi'(a'_{22} + \frac{1}{r}a_{22}) + \Psi\left(\frac{a'_{12}}{r} - \frac{a_{11}}{r^2}\right)$$

$$+ \frac{\gamma}{g}\omega^2 r^2\left(a'_{22} + 3\frac{a_{22}}{r} - \frac{a_{12}}{r}\right) = 0 \ . \qquad (39)$$

Again, for simplicity, the modified orthotropy relations as defined by Equations (24) will be adapted making Equation (39) take the form

$$0 = a'_{22}\left[\Psi' + \frac{k_2}{r}\Psi + \frac{\gamma}{g}\omega^2 r^2\right] + a_{22}\left[\Psi'' + \frac{\Psi'}{r} - \frac{k_1\Psi}{r^2} + \frac{\gamma}{g}\omega^2 r(3-k_2)\right] \qquad (40)$$

The stress criterion will be the same as before, i.e., σ_θ = constant. Using this criterion and operating on the second of Equations (38) together with the assumed boundary conditions that $\sigma_r(a) = \sigma_r(b) = 0$ yields

$$\Psi = \frac{\gamma v^2}{3g}\left(\frac{k}{k-1}\right)\left[r\left(1 - \frac{1}{k^3}\right) - a\left(1 - \frac{1}{k^2}\right) - \left(\frac{r}{b}\right)^3\left(b - a\right)\right]$$

$$\sigma_r = \frac{\gamma v^2}{3g}\left(\frac{k}{k-1}\right)\left[1 - \frac{1}{k^3} - \frac{1}{\rho} + \frac{1}{\rho k^3} - \rho^2 + \frac{\rho^2}{k}\right] \qquad (41)$$

$$\sigma_\theta = \frac{\gamma v^2}{3g}\left(\frac{k}{k-1}\right)\left[\frac{k^3 - 1}{k^3}\right]$$

where $\rho = r/b$

$k = b/a$

$v = \omega b =$ tip velocity

$\omega =$ rotational velocity, radians

$g =$ acceleration of gravity.

Here we note that in the limits, when $k \to 1$

$$\sigma_\theta = \frac{\gamma v^2}{g} \tag{42}$$

which is the stress in a rotating thin ring, and when $k \to \infty$ (i.e., when a very small)

$$\sigma_\theta = \frac{\gamma v^2}{3g}\ , \tag{43}$$

which is smaller than exists for the isotropic, homogeneous case by the ratio

$$\left(\frac{\sigma_{\theta_1}}{\sigma_{\theta_o}}\right)_{max} = \frac{4}{3(3+\nu)} \tag{44}$$

where $\sigma_{\theta_1} =$ hoop stress for heterogeneous case

$\sigma_{\theta_o} =$ hoop stress for isotropic, homogeneous case

$\nu =$ Poisson's ratio for isotropic, homogeneous case .

Substituting Equation (41) into Equation (40) and carrying out the required differentiation yields

$$a'_{22} - \left[\frac{Ar^3 + Br + D}{Fr^4 + Gr^2 + Hr}\right] a_{22} = 0 \tag{45}$$

where $A = (k_1 - 3k_2)$

$B = (1 - k_1)\left[\frac{b^3 - a^3}{b - a}\right]$

$$D = k_1(ab)(b+a)$$

$$F = k_2$$

$$G = -(1+k_2)\ \frac{b^3-a^3}{b-a}$$

$$H = k_2(ab)(b+a)\ .$$

Equation (45) is not readily solvable in closed form. Before proceeding to the solution of Equation (45) by some numerical means, it is convenient to digress at this point to discuss another stress criterion which may be applied to the implementation of design synthesis as defined in this work. This criterion is that through the body of the disk, the in-plane shear stress, τ, is to be a constant. For a body of revolution, acted upon by symmetric loads,

$$\tau = (\sigma_\theta - \sigma_r)/2$$

or

$$\tau = \sigma_\theta - \sigma_r = \text{Constant} = C_o \tag{46}$$

Applying this condition to Equations (38) and integrating leads to

$$\frac{1}{r}\Psi = C_o \ell nr - \frac{\gamma\omega^2 r^2}{2g} + C_1$$

$$\Psi' = C_o \ell nr + C_o - \frac{3}{2}\,\frac{\gamma\omega^2 r^2}{g} + C_1$$

$$\Psi'' = \frac{C_o}{r} - \frac{3}{g}\,\gamma\omega^2 r\ , \tag{47}$$

requiring that on the boundaries;

$$\sigma_r(b) = \sigma_o$$

$$\sigma_r(a) = \sigma_i\ ,$$

leads to the following relations.

$$\sigma_r = \frac{1}{r}\psi = C_o \ln r - \frac{\gamma\omega^2 r^2}{2g} + C_1$$

$$\sigma_\theta = \psi' + \frac{\gamma\omega^2 r^2}{g} = C_o(\ln r + 1) - \frac{\gamma\omega^2 r^2}{2g} + C_1$$

$$C_o = \frac{1}{\ln \frac{b}{a}} \left\{(\sigma_o - \sigma_i) + \frac{\gamma\omega^2}{2g}(b^2 - a^2)\right\}$$

$$C_1 = \frac{1}{\ln \frac{b}{a}} \left\{\sigma_i \ln b - \sigma_o \ln a - \frac{\gamma\omega^2}{2g}(b^2 \ln a - a^2 \ln b)\right\} \qquad (48)$$

These relations, as well as those defined by Equation (41) (σ_θ = constant) were applied to Equation (40) and were solved numerically. A digital computer program, Determination Of Modulus Variation 1 (DOMOV1), was structured for the solution of these sets of equations. A finite difference method of solution as developed by Manson[18] was used for the calculation algorithm. This program, which is detailed in the Appendices was used to solve several problems as follows.

- The modulus variation for an orthotropic disk with a pressurized I.D. for σ_θ = constant (disk not rotating)
- The modulus variation for an isotropic disk with pressurized I.D. for σ_θ = constant (disk not rotating)
- The modulus variation for an isotropic disk with pressurized O.D. for σ_θ = constant (disk not rotating)
- The modulus variation for a rotating orthotropic disk with no edge loads for σ_θ = constant
- The modulus variation for a rotating orthotropic disk (Poisson's ratio variable) with no edge loads for σ_θ = constant
- The modulus variation for a rotating orthotropic disk with no edge load for constant in-plane shear stress
- The modulus variation for a rotating orthotropic disk (Poisson's ratio variable) with no edge loads for constant in-plane shear stress

- The modulus variation for a rotating orthotropic disc with 60.0 inch O.D., 6.0-inch I.D., turning at 2,000 RPM, stressed on the O.D. with an uniform edge load of 10,000 psi for σ_θ = constant
- The modulus variation for a rotating orthotropic disk with 60.0-inch O.D. 6.0-inch I.D., turning at 4,000 RPM, stressed on the O.D. with a uniform edge load of 12,000 psi for σ_θ = constant.

The solutions for the first four problems were checked by use of the closed form solutions, Equations (29) and (33) and are those shown plotted in Figure 2, 3, 4, and 5. The solution to the remaining problems are shown plotted in Figures 7 through 12. It should be noted here that when boundary tractions as well as body loads are applied to a rotating disk, the solution is specific as regards the magnitude of the edge loads and the rotational velocity of the disk. However, where only one type of load is imposed, then the solution is generalized and is independent of the magnitude of the load. (Note: The stresses remain directly dependent on the load.) This solution dependence upon load is shown very clearly by comparing Figures 11 and 12. Here the modulus variation is shown to change with the change of the boundary tractions and the rotational velocity. All these figures have been non-dimensionalized by the expediency of plotting E/E_{max} and R/B where

E = E_θ, the modulus of elasticity in the θ direction.

E_{max} = the maximum E_θ calculated in the body of the disk.

R = the radius of the point in the disk at which the modulus is being calculated.

B = the outer radius of the disk.

The computer output for the curves plotted in Figure 12 is also found here in Appendix C. For this case, σ_θ equals 28,454 psi throughout the disk. For an isotropic, homogenious, flat-angular disk of the same material

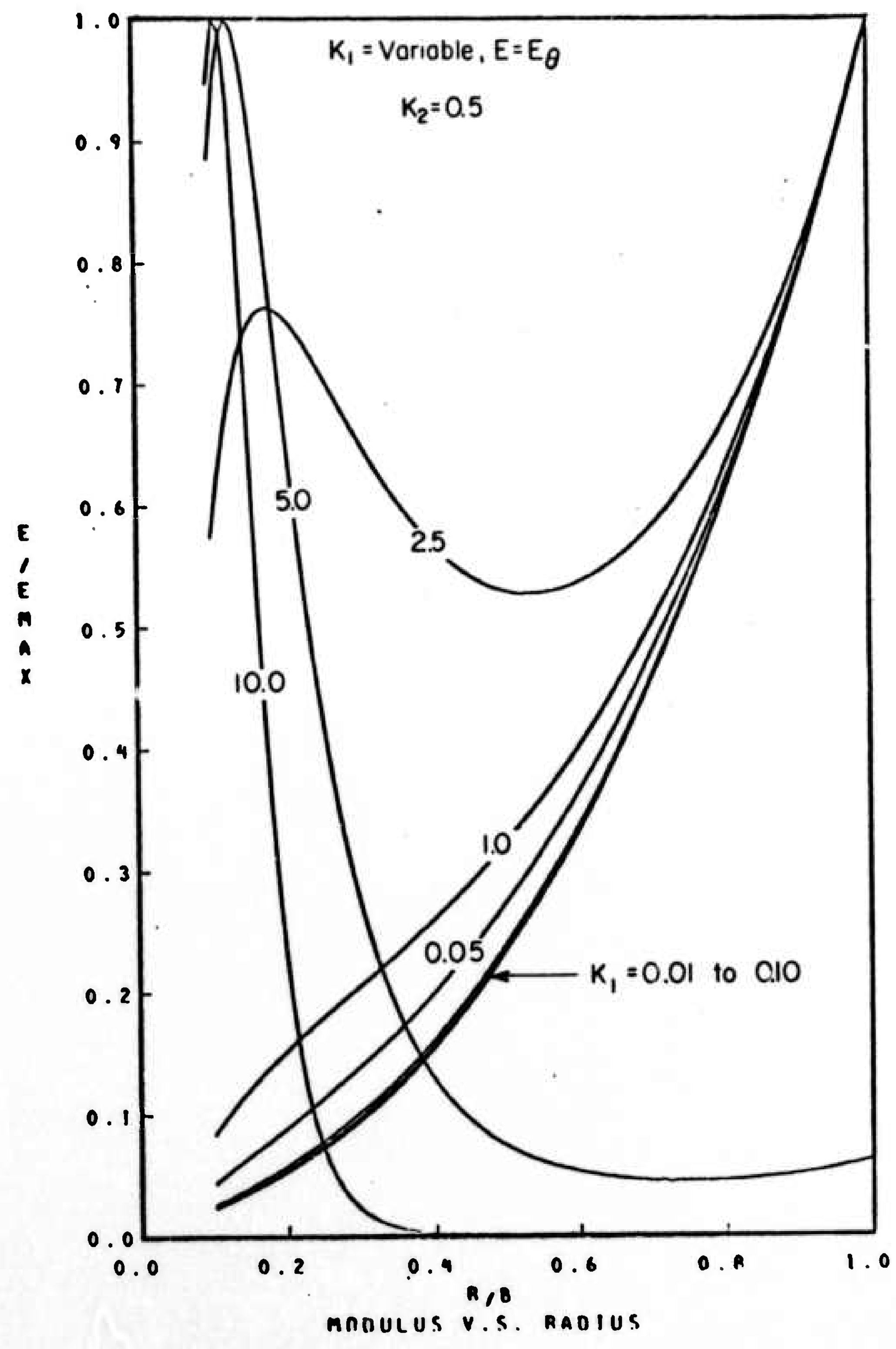

FIGURE 7. MODULUS VARIATION FOR ROTATING ORTHOTROPIC DISK WITH NO EDGE LOADS, WITH σ_θ = CONSTANT

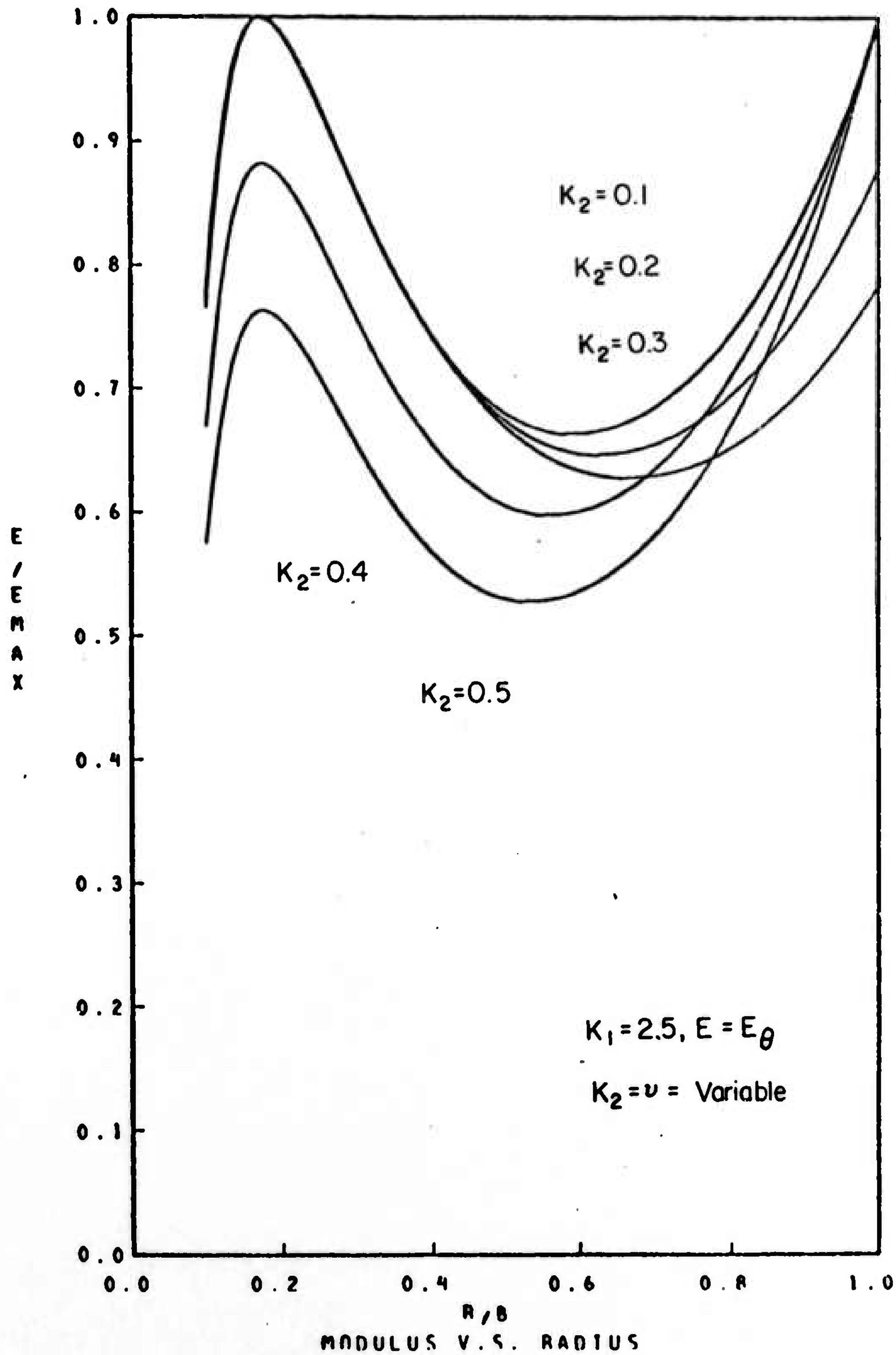

FIGURE 8. MODULUS VARIATION FOR ROTATING ORTHOTROPIC DISK, ORTHOTROPIC RATIO OF 2.5 WITH NO EDGE LOADS AND σ_θ = CONSTANT

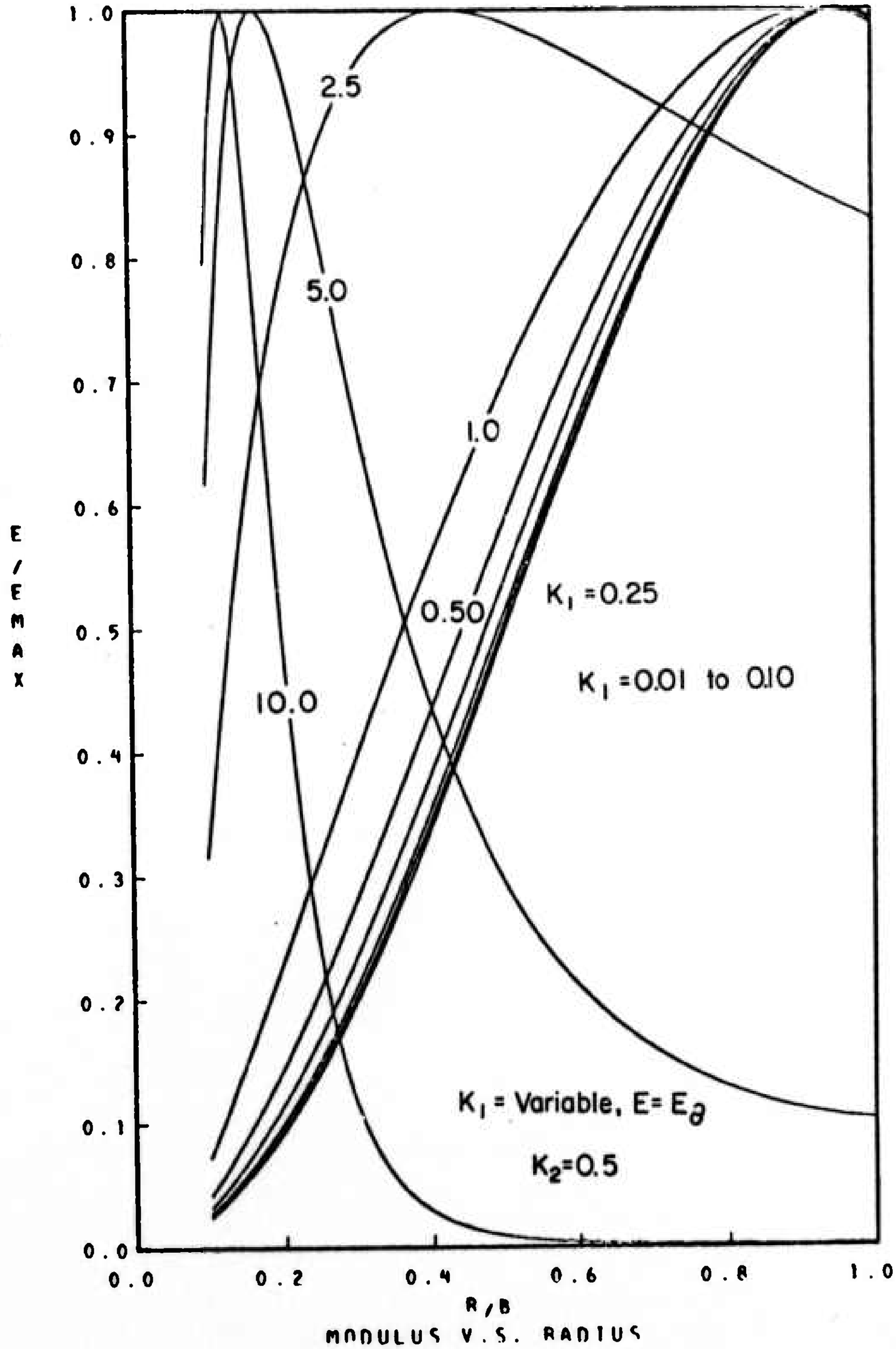

FIGURE 9. MODULUS VARIATION FOR ROTATING ORTHOTROPIC DISK WITH NO EDGE LOADS, AND WITH CONSTANT IN-PLANE SHEAR STRESS

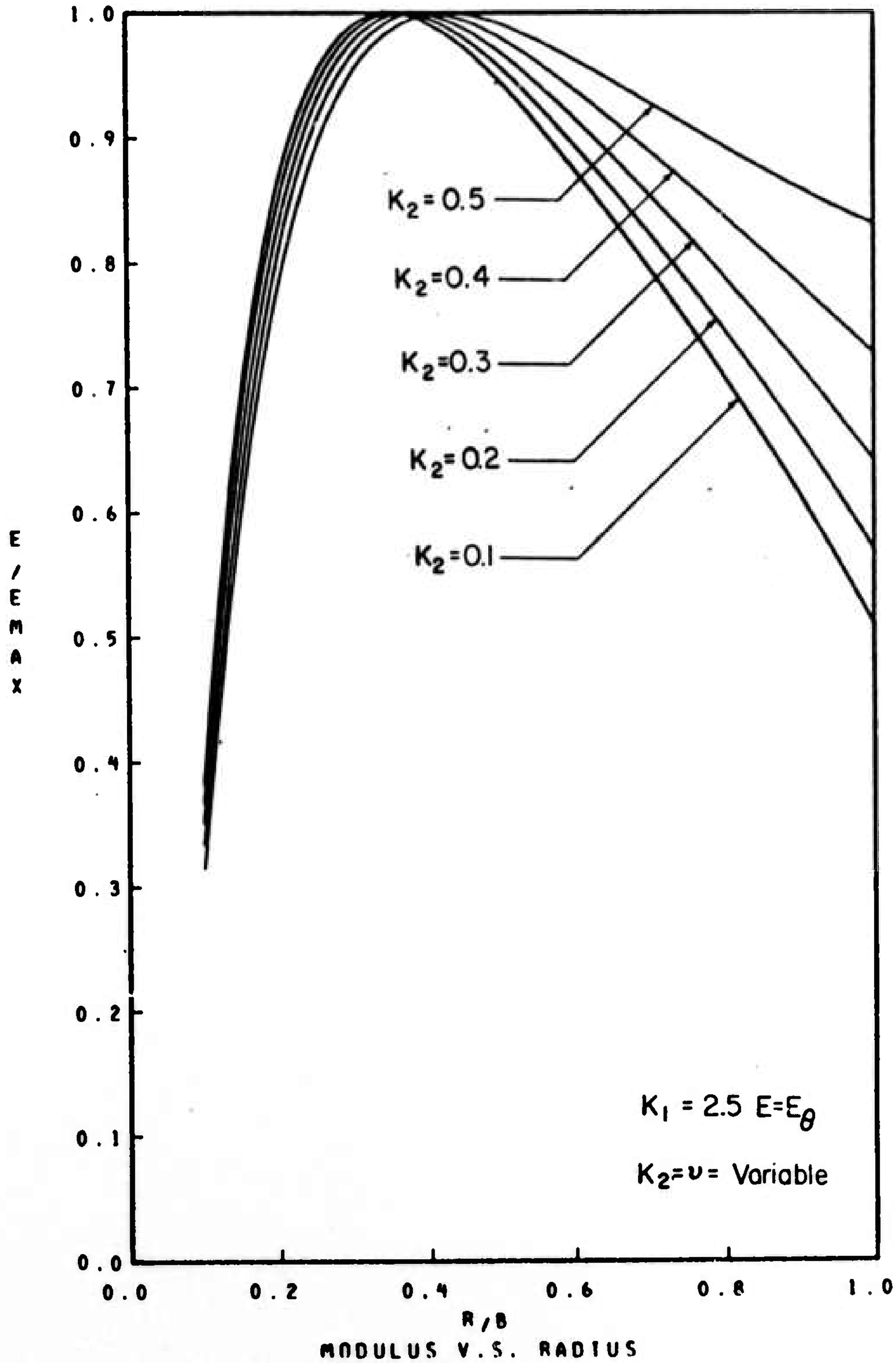

FIGURE 10. MODULUS VARIATION FOR ROTATING ORTHOTROPIC DISK ORTHOTROPIC RATIO OF 2.5 WITH NO EDGE LOADS AND WITH CONSTANT IN-PLANE SHEAR STRESS

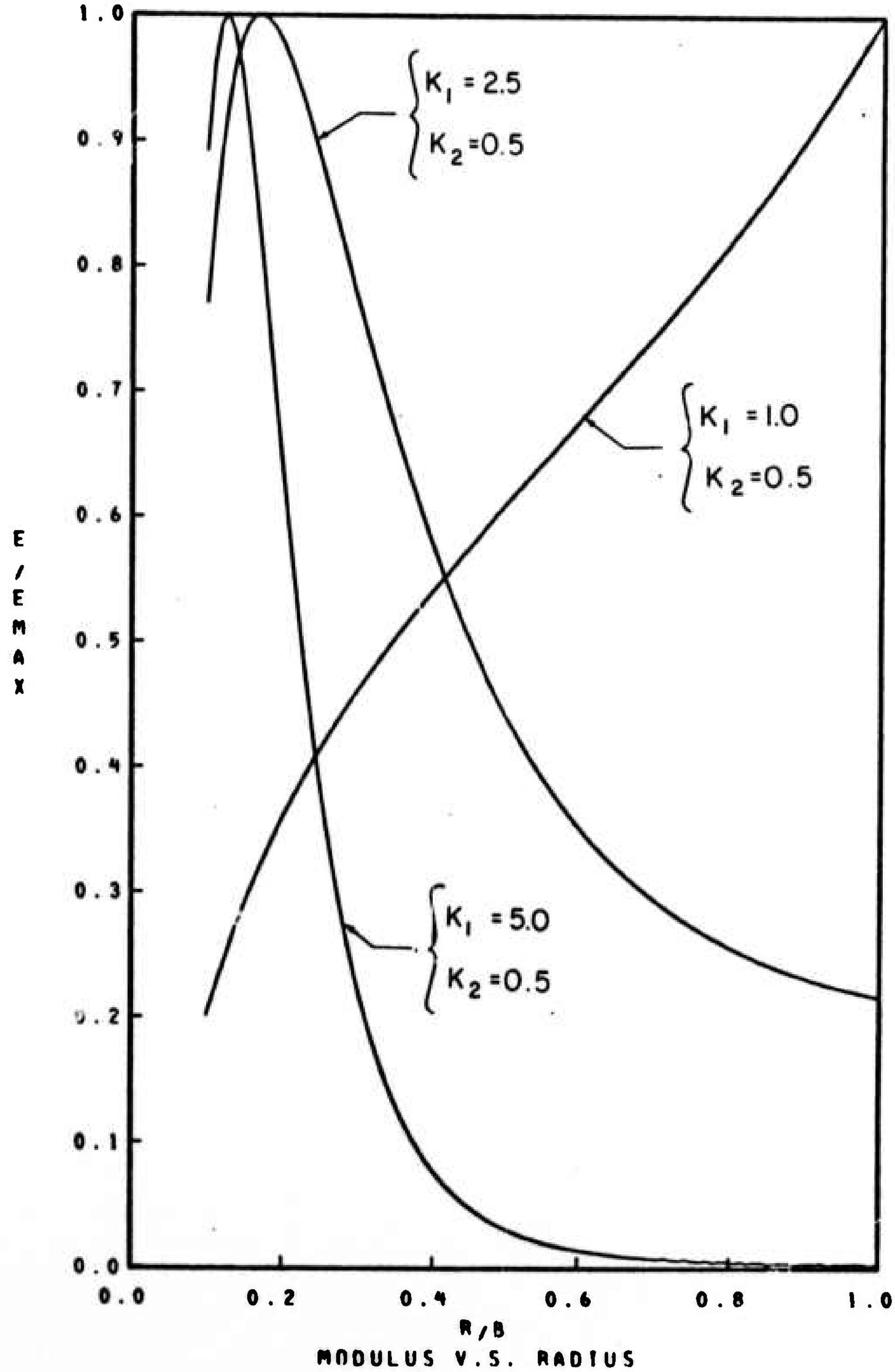

FIGURE 11. MODULUS VARIATION FOR ORTHOTROPIC ROTATING DISK STRESSED ON O.D. RPM = 2000; DENSITY = 0.1 LB/IN.^{3}; STRESS ON O.D. = 10,000 PSI: O.D. = 60.0 INCHES: I.D. = 6.0 INCHES, σ_θ = CONSTANT

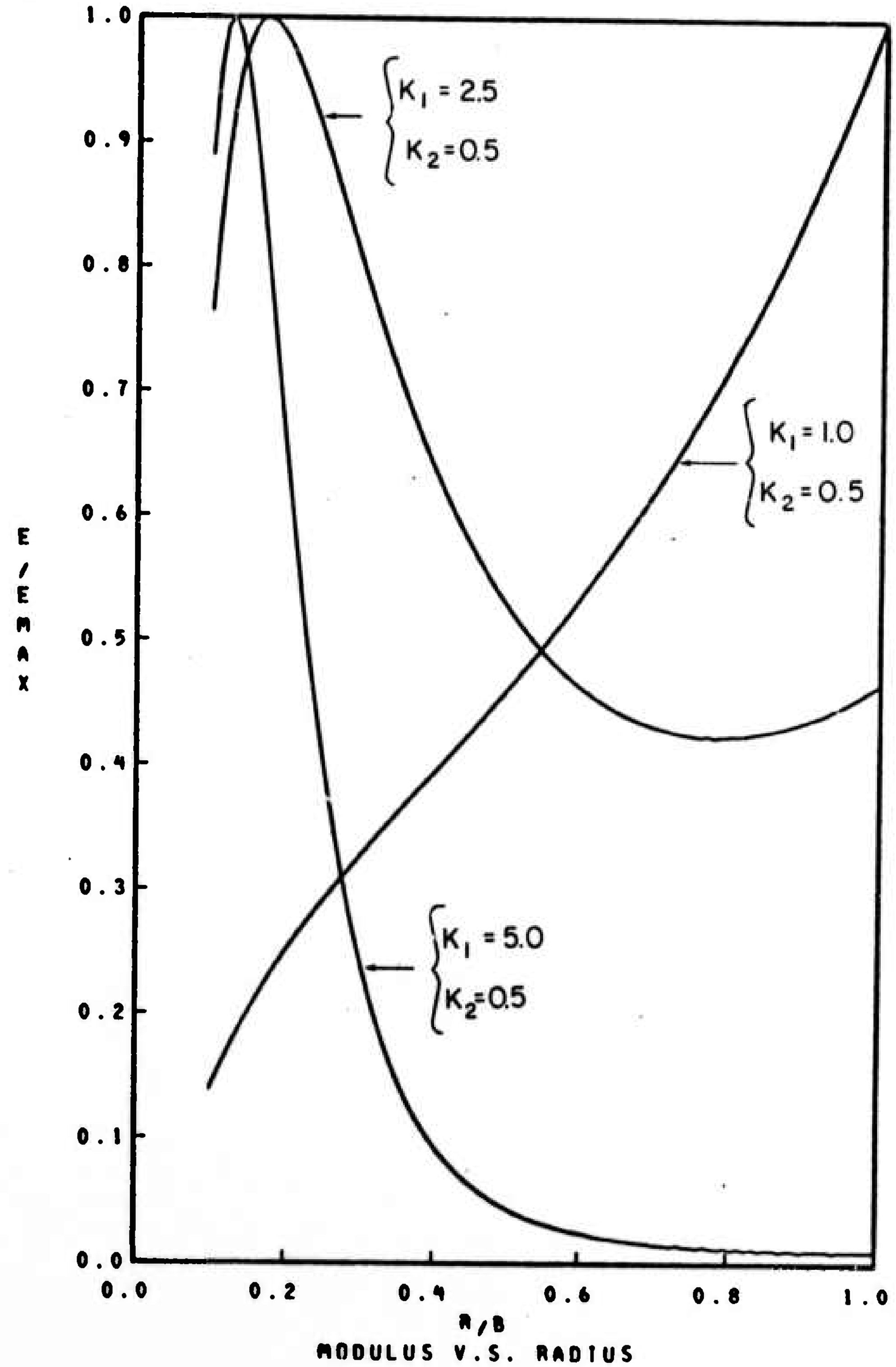

FIGURE 12. MODULUS VARIATION FOR ORTHOTROPIC ROTATING DISK STRESSED ON O.D.: RPM = 4000; DENSITY = 0.1 LB/IN.^{3}; STRESS ON O.D. = 12,000 PSI; O. D. = 60.0 INCHES; I.D. = 6.0 INCHES, σ_θ = CONSTANT

density σ_θ would equal

$$\sigma_\theta = \frac{3+\nu}{4} \rho\omega^2 (b^2 + \frac{1-\nu}{3+\nu} a^2) + 2\sigma_o \frac{b^2}{b^2-a^2} \tag{49}$$

$$\sigma_\theta = 35,810 + 24,242 = 60,052 \text{ psi.}$$

Thus, by the use of design synthesis, the maximum stress in such a disk can be reduced by a factor of 2.11. Further, Figure 12 shows that this can be achieved with a modulus variation through the disk of approximately 2.5 to 1 for a material that exhibits and orthotropic ratio k_1 of 2.5 (the similarity of the two ratios is coincidental and bears no significance).

Non-Symmetric Problems

Example 4: Small hole in an infinite plate. Figure 13 represents a small hole in an infinite plate which is subjected to a uniform tensile stress, P, in the x-direction. For the homogeneous, isotropic condition, the stress distribution around the hole is well known as given by Timoshenko [15] as

$$\sigma_r = \frac{P}{2}\left(1 - \frac{a^2}{r^2}\right) + \frac{P}{2}\left(1 + \frac{3a^4}{r^4} - \frac{4a^2}{r^2}\right) \cos 2\theta$$

$$\sigma_\theta = \frac{P}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{P}{2}\left(1 + \frac{3a^4}{r^4}\right) \cos 2\theta \tag{50}$$

$$\sigma_{r\theta} = -\frac{P}{2}\left(1 - \frac{3a^4}{r^4} + \frac{2a^2}{r^2}\right) \sin 2\theta \quad .$$

The maximum stress occurs at $r = a$, $\theta = (\pi/2, 3\pi/2)$, and is

$$\sigma_{max} = (\sigma_\theta)_{r=0,\theta=\pi/2} = 3P \; .$$

For the homogeneous, anisotropic condition work by Green and Zerna [19], Hearmon [20], Savin [21], Leckhniskii [12], and (as directly applied to composites) by Greszczuk [22], shows that the maximum stress at the hole is usually greater than for the isotropic case and can reach values as high as 9P.

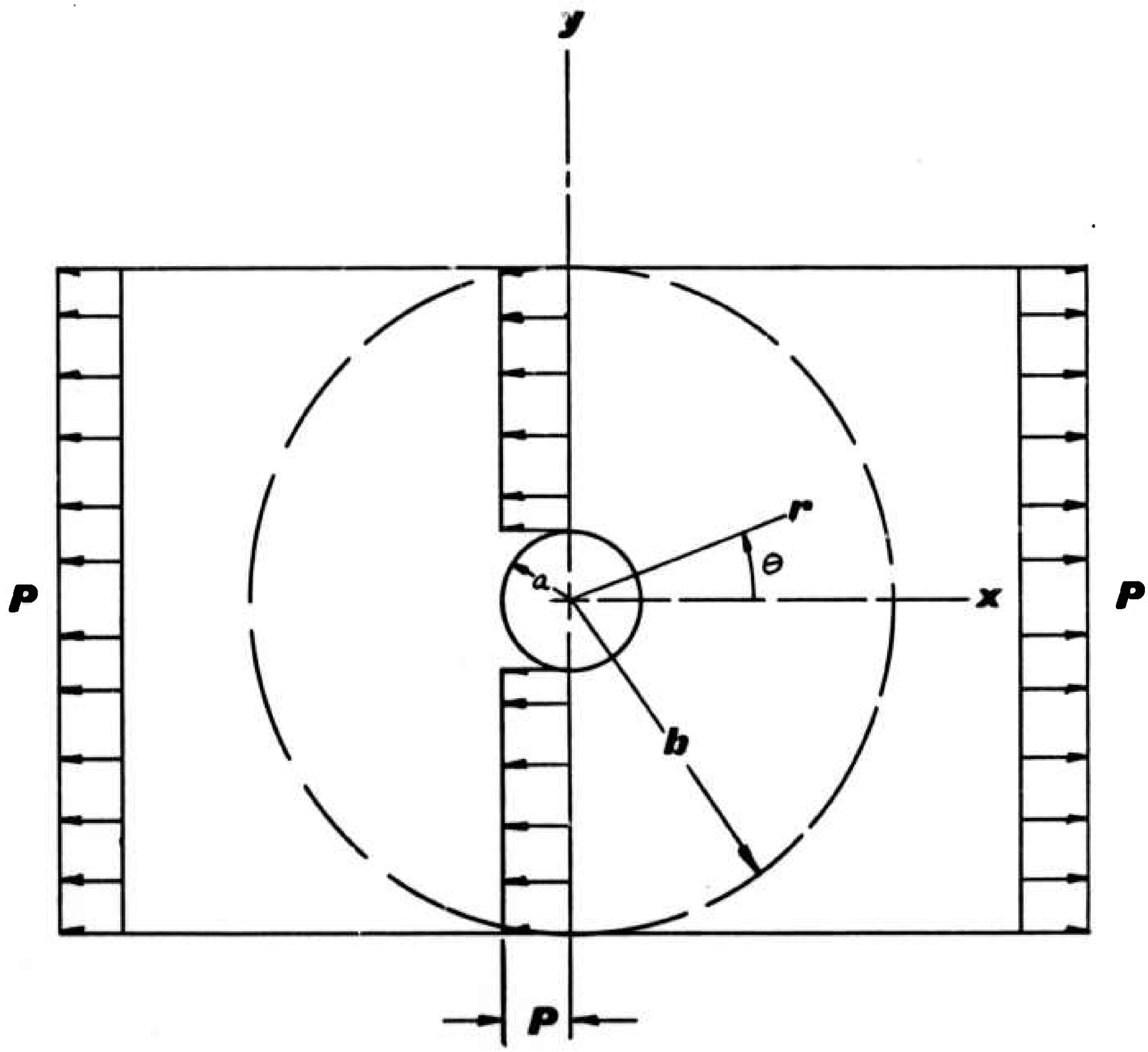

FIGURE 13. SMALL HOLE IN INFINITE PLATE SUBJECTED TO UNIFORM, UNIAXIAL TENSILE STRESS FIELD

Referring to Figure 13, consider the portion of the plate within a concentric circle of radius b, large in comparison with a. It can safely be assumed that the stresses at radius b are effectively the same as in a plate without the hole and can be given by

$$(\sigma_r)_{r=b} \approx \frac{1}{2} P (1 + \cos 2\theta)$$

$$(\sigma_{r\theta})_{r=b} \approx - \frac{1}{2} P \sin 2\theta \ . \tag{51}$$

From Equation (50) it can be seen that

$$(\sigma_\theta)_{r=b} \approx \frac{1}{2} P (1 - \cos 2\theta).$$

It seems reasonable, then, to choose a stress criterion for the plate

$$\sigma_\theta = \text{function of } \theta = \frac{1}{2} P (1 - \cos 2\theta). \tag{52}$$

From the second of Equations (11), with V equal to zero the stress function becomes

$$\Psi = \frac{1}{4} Pr^2 (1 - \cos 2\theta) + f_1(\theta)r + f_2(\theta) \ . \tag{53}$$

where $f_1(\theta)$ and $f_2(\theta)$ are functions of θ only. Applying the first and third of Equations (11) to Ψ results in

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \Psi}{\partial \theta}\right) = - \sigma_{r\theta} = \frac{1}{2} P \sin 2\theta - \frac{1}{r^2} \cdot \frac{df_2(\theta)}{d\theta}$$

$$\frac{1}{r}\frac{\partial \Psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Psi}{\partial \theta^2} = \sigma_r = \frac{1}{2} P (1 + \cos 2\theta) + \frac{1}{r}\frac{d^2 f_1(\theta)}{\alpha r d\theta^2} + \frac{1}{r^2}\frac{d^2 f_2(\theta)}{d\theta^2} + \frac{1}{r} f_1(\theta) \tag{54}$$

Applying the boundary conditions

$$(\sigma_r)_{r=a} = (\sigma_{r\theta})_{r=a} = 0$$

yields

$$f_2(\theta) = -\frac{Pa^2}{4}\cos 2\theta$$

$$\frac{d^2 f_1(\theta)}{d\theta^2} + f_1(\theta) = -\frac{P}{2}\left[1 + 3\cos 2\theta\right] . \tag{55}$$

Choosing as a particular solution to the second of Equations (55)

$$f_1(\theta) = C_1 + C_2 \cos 2\theta \tag{56}$$

yields

$$f_1(\theta) = -\frac{Pa}{2}\left[1 - \cos 2\theta\right]$$

and results in

$$\Psi = \frac{P}{4}\left\{(r^2 - 2ar) - (r - a)^2 \cos 2\theta\right\}$$

$$\sigma_r = \frac{P}{2}\left(1 - \frac{a}{r}\right)\left[1 - \cos 2\theta + \left(1 - \frac{a}{r}\right)\cos 2\theta\right]$$

$$\sigma_\theta = \frac{P}{2}(1 - \cos 2\theta) \tag{57}$$

$$\sigma_{r\theta} = -\frac{P}{2}\left(1 - \frac{a^2}{r^2}\right)\sin 2\theta .$$

The stress function, Ψ, as defined by the first of Equations (57) was applied to Equation (10) with the body functions and temperature difference taken as zero. In order to implement a solution, the following relationships among the material coefficients were assumed.

$$a_{11} = k_1 a_{22}$$

$$a_{22} = a_{22}$$

$$a_{12} = k_2 a_{22}$$

$$a_{66} = k_3 a_{22} \quad .$$

A finite-difference algorithm was structured to carry out the solution. The method of solution applied was that usually referred to as the "relaxation method" which is discussed in detail by Shaw[23], Hildebrand[25], Allen[26], and Richtmyer[27].

Due to symmetry, only one-fourth of the plate was modeled. A square mesh was used and the quarter plate separated into a square array of 61 x 61 nodes. It was assumed that the circular hole has a radius of 1.5 inches and the mesh distance, h, the distance between nodes, is 0.25 inches. The width of the quarter plate model, thus became 15.0 inches. This gave a b to a ratio (see Figure 13) of 10:1, thus minimizing the far-field effects of the outer boundaries upon the stress around the hole. This plate model is shown in Figure 14. Rectangular coordinates were employed and the differential equation that was differenced was that shown in Equation (9). Equations (57) were converted to rectangular coordinates for input to this program. In the solution procedure, it is necessary to assume the values of the modulus parameter a_{22} at all boundary nodes. The solutions were then to proceed in an iterative manner until the values for a_{22} were determined at each interior node in the model. No success was attained by this method. The program never was able to converge to a solution. In fact, a strongly divergent tendency was noted (i.e., each iteration on the a_{22}'s at each node point was markedly greater

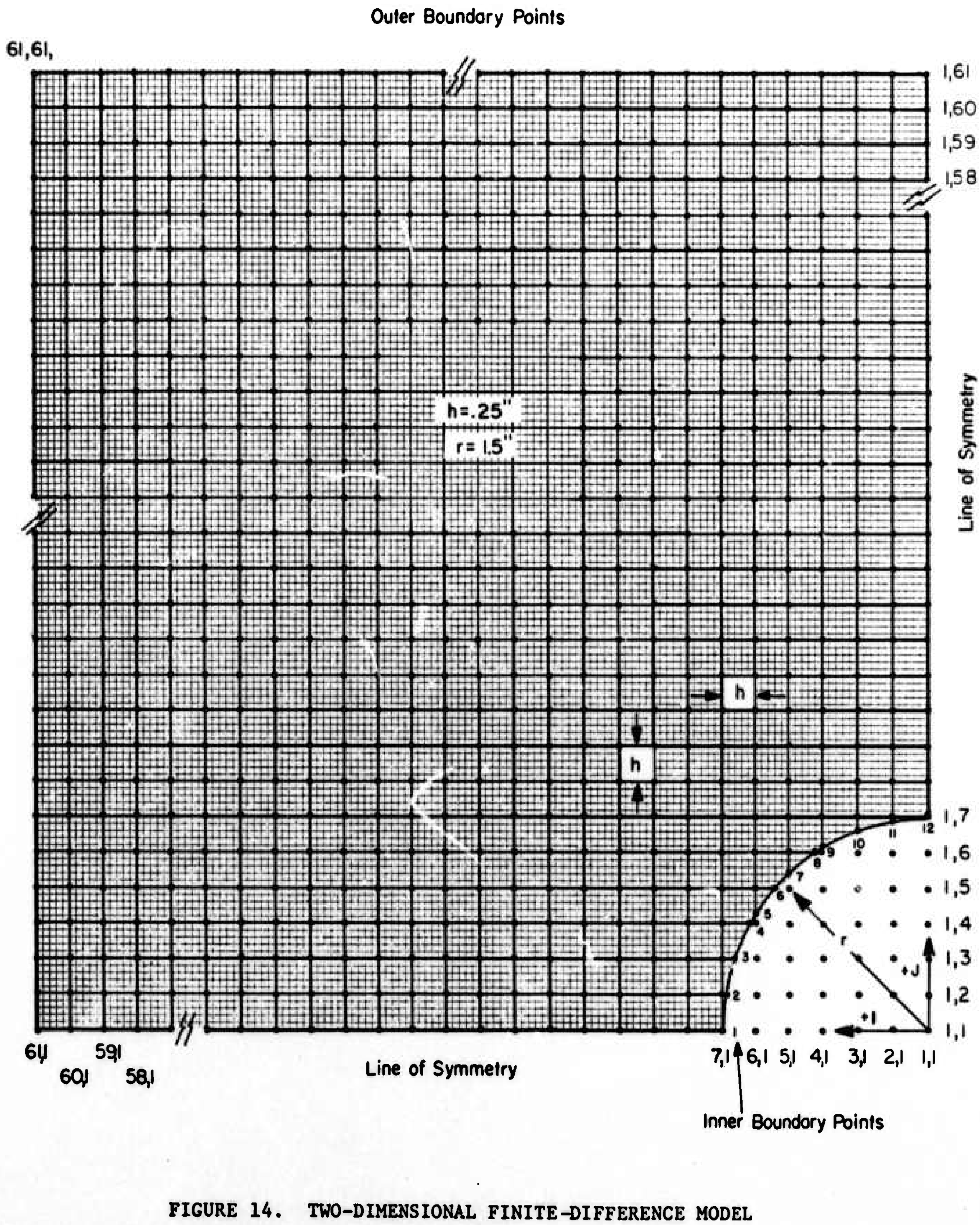

FIGURE 14. TWO-DIMENSIONAL FINITE-DIFFERENCE MODEL

on each iteration in the relaxation process than the iteration which it succeeded, regardless of the boundary values assumed.) Extensive investigations indicated that there was no error in the solution process. It appears that there must be some other governing relations which have as yet not been expressed.

As a result, no solutions to the two-dimensional problem in design synthesis have as yet been accomplished.

Limits and Other Considerations

In the preceeding discussion, it was pointed out that a solution for the two-dimensional problem could not be achieved. This raised questions as to whether all governing equations have been generated. In the application of the one-dimensional program, DOMOV1, it was found that under certain conditions, the only mathematical solution developed for the modulus variation in a disk would require that at some points in the disk body, the value of the modulus must be negative. Though this is perfectly acceptable from a mathematical viewpoint, it obviously cannot be implemented from the viewpoint of real materials. To illustrate this, consider the case of the rotating annular disk with boundary tractions when subject to the stress criterion that the in-plane shear stresses throughout the disk must be constant. From Equations (48)

$$\sigma_o = C_o(\ell n r + 1) - \frac{\gamma\omega^2 r^2}{2g} + C_1$$

$$\sigma_r = C_o \ell n r - \frac{\gamma^2 \omega^2 y^2}{2g} + C_1$$

$$C_o = \frac{1}{\ell n \frac{b}{a}} \left\{(\sigma_o - \sigma_i) + \frac{\gamma\omega^2}{2g}(b^2 - a^2)\right\} \qquad (48)$$

$$C_1 = \frac{1}{\ell n \frac{b}{a}} \left\{\sigma_i \ell n b - \sigma_o \ell n a - \frac{\gamma\omega^2}{2g}(b^2 \ell n a - a^2 \ell n b)\right\}.$$

Take the condition where the disk is not rotating; i.e., $\omega = 0$, then

$$C_o = \frac{\sigma_o - \sigma_i}{\ell n \frac{b}{a}}$$

$$C_1 = \frac{\sigma_i \ell n b - \sigma_o \ell n a}{\ell n \frac{b}{a}} \qquad (59)$$

$$\sigma_\theta = C_o(\ell n r + 1) + C_1$$

$$\sigma_r = C_o \ell n r + C_1 \,. \qquad (60)$$

Substituting Equation (59) in (60) yields

$$\sigma_\theta = \frac{1}{\ln \frac{b}{a}} \left\{ (\sigma_o - \sigma_i)(\ln r + 1) + \sigma_i \ln b - \sigma_o \ln a \right\}$$

$$\sigma_r = \frac{1}{\ln \frac{b}{a}} \left\{ (\sigma_o - \sigma_i)\ln r + \sigma_i \ln b - \sigma_o \ln a \right\} \quad . \qquad (61)$$

Now, further assume that $\sigma_o = 0$, $\sigma_i = -P$. Then, Equation (61) become

$$\sigma_\theta = \frac{P}{\ln \frac{b}{a}} \left\{ 1 - \ln \frac{b}{r} \right\}$$

$$\sigma_r = - \frac{P}{\ln \frac{b}{a}} \left\{ \ln \frac{b}{r} \right\}. \qquad (62)$$

Note that for all values of $r \leq b$, $\sigma_r \leq -P$ in the algebraic sense. However, if $\frac{b}{a} > e$, (2.71828), then at $r = a$, $\ln \frac{b}{a} > 1$ and σ_θ is negative. Defining the displacement at $r = a$ as

$$U_{(r=a)} = \left(\frac{1}{E(\theta)} [\sigma_\theta - \nu\sigma_r] \right) \Big|_{r=a}$$

$$U_{(r=a)} = \frac{1}{E(\theta)} \left\{ \frac{P}{\ln \frac{b}{a}} [1 - \ln \frac{b}{a} + \nu \ln \frac{b}{a}] \right\}$$

$$U_{(r=a)} = \frac{1}{E(\theta)} \left\{ \frac{P}{\ln \frac{b}{a}} [1 - \ln \frac{b}{a} \cdot (1-\nu)] \right\} \quad . \qquad (63)$$

From Equation (63), if $\ln \frac{b}{r} > \frac{1}{1-\nu}$ then the term in the brackets is negative and U or $E(\theta)$ must be negative. If either is negative, then the system must do negative work, which is not possible for real materials. This, of course, can be overcome in a numerical sense by simply requiring that only those solutions are acceptable which yield a material coefficient matrix that is positive definite at each point in the body; i.e., the following conditions must all be met:

$$a_{11} \geq 0$$

$$a_{11}a_{12} - a_{12}^2 \geq 0 \qquad (64)$$

$$a_{11}a_{22}a_{66} - a_{12}^2 a_{66} \geq 0 \quad .$$

Thus, it is clear that the "arbitrariness" of any stress conditions selected are restricted to more than conforming to the equilibrium equations and the boundary conditions.

Two Dimensional Boundary Considerations

It is instructive to approach this problem from the viewpoint of the calculus of variations. Consider the total strain energy in a stretched plate of unit thickness:

$$U = \iint_A U_o \, dxdy \ , \qquad (65)$$

where

$$U_o = \text{strain density at a point}$$

$$= \frac{1}{2} [\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \tau_{xy} \gamma_{xy}] \ .$$

Assume a Hookien material, neglecting time and temperature effects,

$$\varepsilon_x = a_{11}\sigma_x + a_{12}\sigma_y$$

$$\varepsilon_y = a_{12}\sigma_x + a_{22}\sigma_y \qquad (66)$$

$$\gamma_{xy} = a_{66}\tau_{xy} \quad ,$$

and a stress function

$$\sigma_x = \frac{\partial^2\Psi}{\partial y^2}$$

$$\sigma_y = \frac{\partial^2\Psi}{\partial x^2} \qquad (67)$$

$$\gamma_{xy} = -\frac{\partial^2\Psi}{\partial x\partial y} \quad ,$$

Such that Equation (65) becomes

$$U = \iint \frac{1}{2}\left[a_{11}\left(\frac{\partial^2\Psi}{\partial y^2}\right)^2 + 2a_{12}\left(\frac{\partial^2\Psi}{\partial x^2}\right)\left(\frac{\partial^2\Psi}{\partial y^2}\right) + a_{22}\left(\frac{\partial^2\Psi}{\partial x^2}\right)^2 + a_{66}\left(\frac{\partial^2\Psi}{\partial x\partial y}\right)^2\right]dxdy \quad . \qquad (68)$$

Following the method of Weinstock[28], the extremization of (68) is affected by forming the integral $I(\delta)$ by replacing Ψ in the integral of (68) by

$$\phi = \Psi(x,y) + \delta\eta(x,y) \quad , \qquad (69)$$

where $\Psi(x,y)$ is the actual extremizing function and $\eta(x,y)$ is an arbitrary function that is twice continuously differentiable. Then, the integral $I(\delta)$ is an extremum for $\delta = 0$, so that

$$I'(0) = 0 \ . \qquad (70)$$

Writing $f(\Psi_{xx}, \Psi_{yy}, \Psi_{xy})$ = the integrand of Equation (68), and here the subscripts refer to differentiation with respect to x and y. Then, according to (68) and using (69) to compute

$$\frac{\partial\Psi_{xx}}{\partial\delta} = \eta_{xx}, \ \frac{\partial\Psi_{yy}}{\partial\delta} = \eta_{yy}, \ \frac{\partial\Psi_{xy}}{\partial\delta} = \eta_{xy} \ , \qquad (71)$$

and I'(δ) is formed and δ is set to zero, resulting in

$$I'(0) = \iint_A \left(\frac{\partial f}{\partial \Psi_{xx}} \eta_{xx} + \frac{\partial f}{\partial \Psi_{yy}} \eta_{yy} + \frac{\partial f}{\partial \Psi_{xy}} \eta_{xy}\right) dxdy = 0, \tag{72}$$

according to Equation (70). Integrating by parts and employing Green's theorem results in the transformation of Equation (72) to

$$\iint_A \left\{ \eta \left[\frac{\partial^2}{\partial x^2}\left(\frac{\partial f}{\partial \Psi_{xx}}\right) + \frac{\partial^2}{\partial y^2}\left(\frac{\partial f}{\partial \Psi_{yy}}\right) + \frac{\partial^2}{\partial x \partial y} + \left(\frac{\partial f}{\partial \Psi_{xy}}\right)\right]\right\} dxdy$$

$$+ \int_c \left\{ \eta \left[\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial \Psi_{yy}}\right) + \frac{1}{2}\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial \Psi_{xy}}\right)\right] - \eta_y \frac{\partial f}{\partial \Psi_{yy}} - \frac{1}{2}\eta_x \frac{\partial f}{\partial \Psi_{yy}} \right\} dx$$

$$+ \int_c \left\{ \eta_x \frac{\partial f}{\partial \Psi_{xx}} + \frac{1}{2}\eta_y \frac{\partial f}{\partial \Psi_{xy}} - \eta \left[\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) + \frac{1}{2}\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial \Psi_{xy}}\right)\right]\right\} dy$$

$$= 0 . \tag{73}$$

Noting that

$$\frac{\partial f}{\partial \Psi_{xx}} = a_{12}\left(\frac{\partial^2 \Psi}{\partial y^2}\right) + a_{22}\left(\frac{\partial^2 \Psi}{\partial x^2}\right) = \varepsilon_y$$

$$\frac{\partial f}{\partial \Psi_{yy}} = a_{11}\left(\frac{\partial^2 \Psi}{\partial y^2}\right) + a_{12}\left(\frac{\partial^2 \Psi}{\partial x^2}\right) = \varepsilon_x \tag{74}$$

$$\frac{\partial f}{\partial \Psi_{xy}} = a_{66}\left(\frac{\partial^2 \Psi}{\partial x \partial y}\right) = -\gamma_{xy} ,$$

(Note: Confusing subscripts. Subscripts on strains do not refer to differentiation.)

thus, the area integral becomes

$$\iint_A \left\{ \eta \left[\frac{\partial^2 \varepsilon_y}{\partial x^2} + \frac{\partial^2 \varepsilon_x}{\partial y^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}\right]\right\} dxdy . \tag{75}$$

Since (73) must hold for arbitrary η, it must in particular hold for those η for which $\eta = \eta_x = \eta_y = 0$ on the boundary C. For such η Equation (73) reduces to the well-known compatibility equation as previously derived (see Equation (3)) and the resulting (Equation (9)). For arbitrary η, η_x, η_y, other than zero, the boundary relations must be derived from the remaining line integral portion of Equation (73). The first part of the line integral becomes, along C on y = constant,

$$\eta\left[\frac{\partial a_{11}}{\partial y}\frac{\partial^2\psi}{2y^2} + a_{11}\frac{\partial^3\psi}{\partial y^3} + \frac{\partial a_{12}}{\partial y}\frac{\partial^2\psi}{\partial x^2} + (a_{12} + \frac{1}{2}a_{66})\frac{\partial^3\psi}{\partial x^2\partial y} + \frac{1}{2}\frac{\partial a_{66}}{\partial x}\frac{\partial^2\psi}{\partial x\partial y}\right]$$

$$- \eta_y\left[a_{11}\frac{\partial^2\psi}{\partial y^2} + a_{12}\frac{\partial^2\psi}{\partial x^2}\right] - \eta_x\,\frac{1}{2}\left[a_{66}\frac{\partial^2\psi}{\partial x\partial y}\right] = 0. \qquad (76)$$

The last two terms in Equation (76) are

$$- \eta_y\left[\epsilon_x\right] - \left[\eta_x\,\frac{1}{2}\gamma_{xy}\right] . \qquad (77)$$

If these strains are not prescribed as zero, then along this portion of the boundary $\eta_x = \eta_y = 0$, and the boundary equation which must be satisfied is

$$\frac{\partial a_{11}}{\partial y}\frac{\partial^2\psi}{\partial y^2} + a_{11}\frac{\partial^3\psi}{\partial y^3} + \frac{\partial a_{12}}{\partial y}\frac{\partial^2\psi}{\partial x^2} + (a_{12} + \frac{1}{2}a_{66})\frac{\partial\psi}{\partial x^2\partial y}$$

$$+ \frac{1}{2}\frac{\partial a_{66}}{\partial x}\frac{\partial^2\psi}{\partial x\partial y} = 0, \qquad (78)$$

and a similar set of relations can be written for the second part of the line integral. These equations have, at best, a very limited application other than showing that other constraints do exist on the boundary. Their limited application is due in great part to the choice of a rectangular coordinate system. All attempts to date to express these relations in terms of other coordinate systems, in particular those using generalized normal and tangential components have been unsuccessful. More work must be done along these lines.

Material Considerations

Finally, just a brief note on the possibility of achieving a material with tailorable properties. It is sufficient here to say that work by Adams and Tsai[29], Dimmock and Abrahams[30], Hewitt and Malherbe[31] Halpin[32], Halpin and Pagano[33], Kohn and Krumhansl[34] Tabaddor[35], Fotinich[36], Wang[37], and Fokin and Shermergor[38], among others, have clearly established that material properties for composites of various types can be established by a knowledge of the known properties of the constituent materials, their orientation and their relative density.

SUMMARY

Mathematical design synthesis has been shown to be possible in certain specific applications. The selection of a design criterion in the cases discussed, two dealing with stress distributions, and the development of the material property distribution within a plane body such that compatibility is satisfied, appears to be a rational basis of design for composite materials. Difficulty has been encountered in the solution of two-dimensional problems employing this concept due to the, as yet, undefined boundary restraint requirements which affects the selection of the stress criterion to be met.

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APPENDIX A-1

INPUT DATA FOR PROGRAM DOMOV1

Five cards constitute the input data for DOMOV1 for each variation to be studied. These five cards represent three (3) data sets. As many runs may be stacked behind the initial set as required. The computer run will terminate when an End of File card is read.

Data Set 1 (Title Cards)

```
READ (5, 10) (ITIL(I, 1), I = 1, 24)
10 Format (8A10)
```

Any information can be placed on these three cards. It is necessary to have three cards, although any can be left blank.

Data Set 2

```
READ (5, 20) RO, RI, RPM, DENS, N, IBOND, IST
20 Format (4F 15.0, 3I15)
```

RO = Outer Radius, Inches

RI = Inner Radius, Inches

RPM = Revolution per minute of Disk

DENS = Material Density of Disk, $lb/in.^3$

N = Finite Difference Increment and Printout Number

IBOND = Type of Boundary Condition on Inner Radius

- 0 = A stress condition
- 1 = A displacement condition

IST = Type of Stress Criterion Chosen

- 1 = Constant theta stress
- 2 = Constant in-Plane shear stress

Data Set 3

If IBOND Equals 0,

READ (5, 40) SI, SO, ORTHO, PO, ETHETA

40 Format (2F 10.0, 3F 15.0)

SI	=	Radial Stress on Inner Radius
SO	=	Radial Stress on Outer Radius
ORTHO	=	Orthotropic Ratio, E_r/E_θ
PO	=	Poisson's Ratio
ETHETA	=	Modulus of Elasticity in Theta Direction at the Inner Radius .

If IBOND Equals 1

READ (5, 30) UI, SO, ORTHO, PO, ETHETA

30 Format (2F 10.0, 3F 15.0)

UI = Selected Radial Displacement of Inner Radius .

APPENDIX B

LISTING OF PROGRAM DOMOV1

```
      PROGRAM DOMOV1 (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
      DIMENSION R(100),ETHET(100),EOC(100),A22(100),CC(100),DD(100),
     $          PHI(100),PHIP(100),PHIPP(100),SIGR(100),SIGT(100),U(100)
     $          ,ROP(100),ITTL(24,1)
      DIMENSION BB(100),AA(100),EE(100)
C
C
      COMMON R,PHI,PHIP,PHIPP,SIGR,SIGT,U
C
C
    5 READ (5,10) (ITTL(I,1),I=1,24)
   10 FORMAT (8A10)
      IF(EOF,5) 120,15
   15 READ (5,20) RO,RI,RPM,DENS,N,IBOND,IST
   20 FORMAT (4F15.0,3I5)
      IF(IBOND-1) 35,25,25
   25 READ (5,30) UI,SO,ORTHO,PO,ETHETA
   30 FORMAT (2F10.0,3F15.0)
C          RO        =OUTER RADIUS,INCHES
C          RI        =INNER RADIUS, INCHES
C          RPM       =REVOLUTIONS PER MINUTE OF DISK
C          DENS      =MATERIAL DENSITY OF DISK, LB./CUBIC INCH
C          N         =FINITE DIFFERENCE INCREMENT NUMBER
C          IBOND     =TYPE OF BOUNDARY CONDITION ON INNER RADIUS
C                     0 = A STRESS CONDITION
C                     1 = A DISPLACEMENT CONDITION
C          IST       =TYPE OF STRESS CRITERION CHOSEN.  PRESENTLY
C                     1 = CONSTANT THETA STRESS
C                     2 = CONSTANT IN-PLANE SHEAR STRESS
C          UI        =RADIAL DISPLACEMENT OF INNER RADIUS
C          SI        =RADIAL STRESS ACTING ON INNER RADIUS
C          SO        =RADIAL STRESS ACTING ON OUTER RADIUS
C          ORTHO     =ORTHOTROPIC RATIO, E-SUB-R/E-SUB-THETA
C          PO        =POISSONS RATIO IN THE THETA-R DIRECTION
C          ETHETA    =MODULUS OF ELASTICITY IN THE THETA DIRECTION AT
C                     THE INNER RADIUS
C          ROB       =RATIO OF THE RADIUS OVER THE OUTER RADIUS
C          EOC       =RATIO OF THE MODULUS OVER THE MAXIMUM MODULUS
C                     FOUND IN THE BODY OF THE DISK
C          PHI       =THE STRESS FUNCTION
C          PHIP      =THE FIRST DERIVATIVE OF THE STRESS FUNCTION
C          PHIPP     =THE SECOND DERIVATIVE OF THE STRESS FUNCTION
C          SIGR      =RADIAL STRESS
C          SIGT      =CIRCUMFERENTIAL STRESS
C          U         =THE RADIAL DISPLACEMENT
C          ETHET     =THE CALCULATED CIRCUMFERENTIAL MODULUS AT EACH
```

```
C                    CALCULATION POINT IN THE DISK
C        A22         =1.0/ETHET
      GO TO 45
   35 READ (5,40) SI,SO,ORTHO,PO,ETHETA
   40 FORMAT (2F10.0,3F15.0)
   45 PO = (-1.0)*PO
      PI=3.141592654
      OMEGA=(RPM*PI)/(30.0)
      GRHO=(DENS)*(OMEGA**2.0)/(386.4)
      A22(1)=1.0/ETHETA
      ETHET(1)=ETHETA
      R(1)=RI
      DEL=(RO-RI)/(N-1)
      DO 50 I=2,N
      J=I-1
      R(I)=R(J)+DEL
   50 CONTINUE
      RA=RO/RI
      CALL STRESS (N,RA,GRHO,SI,SO,UI,ORTHO,PO,ETHETA,IBOND,IST,RI,RO)
      DO 55 I=1,N
      AA(I)=PHIP(I)+(PO*PHI(I))/(R(I))+(GRHO*(R(I)**2.0))
      BB(I)=PHIPP(I)+PHIP(I)/R(I)-(ORTHO*PHI(I))/(R(I)**2.0) +(GRHO*R(I
     $)*(3.0-PO))
      CC(I)=BB(I)/AA(I)
   55 CONTINUE
      DD(1)=0.0
      EE(1)=0.0
      DO 60 I=2,N
      J=I-1
      DD(I)=1.0+CC(I)*(R(I)-R(J))/(2.0)
      EE(I)=1.0-CC(J)*(R(I)-R(J))/2.0
      A22(I)=(EE(I)/DD(I))*(A22(J))
      ETHET(I)= 1.0/A22(I)
   60 CONTINUE
      EMAX=0.0
      DO 65 I=1,N
      EMAX=AMAX1(ETHET(I),EMAX)
   65 CONTINUE
      DO 70 I=1,N
      ROB(I)=R(I)/RO
      EOC(I)=ETHET(I)/EMAX
      U(I)=A22(I)*R(I)*(SIGT(I)+PO*SIGR(I))
   70 CONTINUE
      PO=-1.0*PO
      WRITE (6,75) (ITIL(I,1),I=1,24)
   75 FORMAT (1H1,8A10/1H ,8A10/1H ,8A10///)
      IF(IBOND-1) 80,105,105
   80 WRITE (6,85)RO,RI,RPM,SO,SI,ETHETA,PO,ORTHO,DENS,N
```

```
   85 FORMAT (5X,53HDETERMINATION OF MODULUS VARIATION IN AN ANNULAR DIS
     $K,//,1X,10HINPUT DATA,/,1X,10H**********,//,5X,15HOUTER RADIUS = ,
     $F10.4,2X,15HINNER RADIUS = ,F10.4,2X,6HRPM = ,F10.2,//,5X,30HRADIA
     $L STRESS, OUTER RADIUS = ,F10.2,2X,30HRADIAL STRESS, INNER RADIUS
     $= ,F10.2,//,5X,25HETHETA AT INNER RADIUS = ,0PE15.7,2X,17HPOISSONS
     $ RATIO = ,F10.8,2X,20HORTHOTROPIC RATIO = ,F10.4,//,5X,19HMATERIAL
     $ DENSITY = ,F10.5,2X,26HNUMBER OF RADIAL POINTS = ,I3,///)
   90 WRITE (6,95)
   95 FORMAT (1X,11HOUTPUT DATA,/,1X,11H***********,//,7X,6HRADIUS,9X,6H
     $ETHETA,8X,7HSIGMA-R,8X,7HSIGMA-T,10X,4HR/RO,10X,4HE/EO,10X,7HU-SUB
     $-R,/,7X,6H******,9X,6H******,8X,7H*******,8X,7H*******,10X,4H****,
     $10X,4H****,10X,7H*******,//)
      WRITE (6,100)(R(I),ETHET(I),SIGR(I),SIGT(I),ROR(I),EOC(I),U(I),I=1
     $,N)
  100 FORMAT (5X,F10.4,2X,3PE13.6,8X,F7.0,8X,F7.0,6X,F8.5,6X,F9.4,6X,
     $E12.6)
      GO TO 115
  105 WRITE(6,110) RO,RI,RPM,SO,UI,ETHETA,PO,ORTHO,DENS,N
  110 FORMAT (5X,53HDETERMINATION OF MODULUS VARIATION IN AN ANNULAR DIS
     $K,//,1X,12HINPUT DATA,/,1X,10H**********,//,5X,15HOUTER RADIUS = ,
     $F10.4,2X,15HINNER RADIUS = ,F10.4,2X,6HRPM = ,F10.2,//,5X,30HRADIA
     $L STRESS, OUTER RADIUS = ,F10.2,2X,35HRADIAL DISPLACEMENT, INNER R
     $ADIUS = ,F10.7,//,5X,25HTHETA AT INNER RADIUS = ,0PE15.7,2X,17HPOI
     $SSONS RATIO = ,F10.8,2X,20HORTHOTROPIC RATIO = ,F10.4,//,5X,19HMAT
     $ERIAL DENSITY = ,F10.5,2X,26HNUMBER OF RADIAL POINTS = ,I3,///)
      GO TO 90
  115 CONTINUE
      GO TO 5
  120 CONTINUE
      CALL EXIT
      END
```

```
      SUBROUTINE STRESS(N,RA,GRHO,SI,SO,UI,ORTHO,PO,ETHETA,IBOND,IST,RI,
     $RO)
C
      DIMENSION R(100),ETHET(100),EOC(100),A22(100),CC(100),DD(100),
     $          PHI(100),PHIP(100),PHIPP(100),SIGR(100),SIGT(100),U(100)
     $          ,ROB(100),ITIL(24,1)
      DIMENSION BB(100),AA(100),EE(100)
C
C
      COMMON R,PHI,PHIP,PHIPP,SIGR,SIGT,U
C
C
C        THIS SUBROUTINE CALCULATES THE STRESS FUNCTION AND
C        ITS FIRST AND SECOND DERIVATIVESFOR THE FOLLOWING CONDITIONS,
C        IF IBOND = 0 AND IST = 1 WE HAVE A STRESS CONDITION ON THE
C              INNER RADIUS AND A STRESS CRITERION OF CONSTANT THETA
C              STRESS
C        IF IBOND = 0 AND IST  = 2 WE HAVE A STRESS CONDITION ON THE
C              INNER RADIUS AND A STRESS CRITERION OF CONSTANT IN-PLANE
C              SHEAR STRESS
C        IF IBOND = 1 AND IST = 1 WE HAVE A DISPLACEMENT CONDITION ON
C              THE INNER RADIUS WITH A STRESS CONDTION OF CONSTANT
C              THETA STRESS.
C        THESE ARE THE ONLY CONDITIONS PROGRAMMED
C
      IF(IBOND-1)5,30,30
    5 GO TO (10,20),IST
   10 CONTINUE
C
C        THIS SECTION OF THE SUBROUTINE CONTAINS THE CALCULATIONS
C        FOR  A CONSTANT  CIRCUMFERENTIAL STRESS CONDTION
C
      A0=(1.0/(RO-RI))*((SO*RO-SI*RI)+((GRHO/3.0)*(RO**3.0-RI**3.0)))
      B0=((RO*RI)/(RO-RI))*((SI-SO)-(GRHO/3.0)*(RO**2.0-RI**2.0))
      DO 15 I=1,N
      PHI(I)=A0*R(I)-(GRHO/3.0)*(R(I)**3.0) + B0
      PHIP(I)=A0-(GRHO)*(R(I)**2.0)
      PHIPP(I)=(-2.0)*(GRHO)*R(I)
      SIGR(I)=PHI(I)/R(I)
      SIGT(I)=PHIP(I)+GRHO*(R(I)**2.0)
   15 CONTINUE
      GO TO 40
   20 CONTINUE
C
C        THIS SECTION OF THE SUBROUTINE CONTAINS THE CALCULATIONS
C        FOR A CONSTANT IN-PLANE SHEAR STRESS CRITERION
C
```

```
      C1=(1.0/ALOG(RA))*(SI*ALOG(RO)-SO*ALOG(RI)-(GRHO/2.0)*((RO**2.0)*A
     $LOG(RI)-(RI**2.0)*(ALOG(RO))))
      C2=(1.0/ALOG(RA))*(SO-SI+(GRHO/2.0)*(RO**2.0-RI**2.0))
      DO 25 I=1,N
      PHI(I)=C2*R(I)*ALOG(R(I))-0.5*GRHO*(R(I)**3.0) + C1*R(I)
      PHIP(I)=C2*ALOG(R(I))+C2-1.5*GRHO*(R(I)**2.0)+C1
      PHIPP(I)=C2/R(I)-3.0*GRHO*R(I)
      SIGR(I)=PHI(I)/R(I)
      SIGT(I)=PHIP(I)+GRHO*(R(I)**2.0)
   25 CONTINUE
      GO TO 40
   30 CONTINUE
C
C        THIS SECTION OF THE SUBROUTINE CONTAINS THE CALCULATIONS
C        FOR A DISPLACEMENT CONDITION AT THE INNER RADIUS WITH A
C        STRESS CRITERION OF CONSTANT THETA STRESS
C
      A22(1)=1.0/ETHETA
      A0=((ETHETA)/(RI*(1.0+PO)-PO*RO))*(UI-SO*A22(1)*PO*RO-(GRHO*A22(1)
     $*(1.0/3.0)*PO*(RO**3.0-RI**3.0)))
      B0=SO*RO+(GRHO/3.0)*(RO**3.0)-A0*RO
      DO 35 I=1,N
      PHI(I)=A0*R(I)-(GRHO/3.0)*(R(I)**3.0)+B0
      PHIP(I)=A0-(GRHO)*(R(I)**2.0)
      PHIPP(I)=(-2.0)*(GRHO)*R(I)
      SIGR(I)=PHI(I)/R(I)
      SIGT(I)=PHIP(I)+GRHO*(R(I)**2.0)
   35 CONTINUE
   40 CONTINUE
      RETURN
      END
```

APPENDIX C

OUTPUT OF PROGRAM DOMOV1

ROTATING ANNULAR DISK STRESSED ON O.D.
ORTHOTROPIC RATIO EQUALS 1.0, POISSONS RATIO EQUALS 0.5 .
CONSTANT HOOP STRESS CRITERION

DETERMINATION OF MODULUS VARIATION IN AN ANNULAR DISK

INPUT DATA
****

OUTER RADIUS = 30.0000 INNER RADIUS = 3.0000 RPM = 4000.00

RADIAL STRESS, OUTER RADIUS = 12000.00 RADIAL STRESS, INNER RADIUS = 0.00

ETHETA AT INNER RADIUS = 0.6000000E+07 POISSONS RATIO = 0.5000000 ORTHOTROPIC RATIO = 1.0000

MATERIAL DENSITY = 0.10000 NUMBER OF RADIAL POINTS = 100

OUTPUT DATA
****

RADIUS ******	ETHETA ******	SIGMA-R *******	SIGMA-T *******	R/RO ****	E/EO ****	U-SUB-R *******
3.0000	0.6000000E+07	0	28454.	0.10000	0.1379	0.142272E-01
3.2727	0.6546997E+07	2334.	28454.	0.10909	0.1504	0.136531E-01
3.5455	0.7058808E+07	4303.	28454.	0.11818	0.1623	0.132113E-01
3.8182	0.7554597E+07	5984.	28454.	0.12727	0.1737	0.128691E-01
4.0909	0.8029236E+07	7434.	28454.	0.13636	0.1846	0.126037E-01
4.3636	0.8484322E+07	8697.	28454.	0.14545	0.1950	0.123984E-01
4.6364	0.8922250E+07	9806.	28454.	0.15455	0.2051	0.122412E-01
4.9091	0.9339917E+07	10784.	28454.	0.16364	0.2147	0.121226E-01
5.1818	0.9742887E+07	11653.	28454.	0.17273	0.2239	0.120356E-01
5.4545	0.1013301E+08	12429.	28454.	0.18182	0.2329	0.119751E-01
5.7273	0.1050944E+08	13125.	28454.	0.19091	0.2415	0.119362E-01
6.0000	0.1086605E+08	13750.	28454.	0.20000	0.2498	0.119156E-01
6.2727	0.1121616E+08	14315.	28454.	0.20909	0.2578	0.119104E-01
6.5455	0.1155506E+08	14827.	28454.	0.21818	0.2656	0.119183E-01
6.8182	0.1188553E+08	15291.	28454.	0.22727	0.2732	0.119374E-01
7.0909	0.1220862E+08	15713.	28454.	0.23636	0.2806	0.119660E-01
7.3636	0.1251906E+08	16097.	28454.	0.24545	0.2878	0.120028E-01
7.6364	0.1282244E+08	16447.	28454.	0.25455	0.2948	0.120467E-01
7.9091	0.1312335E+08	16766.	28454.	0.26364	0.3017	0.120965E-01
8.1818	0.1341616E+08	17058.	28454.	0.27273	0.3084	0.121516E-01
8.4545	0.1370306E+08	17324.	28454.	0.28182	0.3150	0.122111E-01
8.7273	0.1398036E+08	17567.	28454.	0.29091	0.3215	0.122744E-01
9.0000	0.1422647E+08	17789.	28454.	0.30000	0.3279	0.123409E-01
9.2727	0.1453394E+08	17991.	28454.	0.30909	0.3342	0.124102E-01
9.5455	0.1481008E+08	18175.	28454.	0.31818	0.3405	0.124818E-01
9.8182	0.1507794E+08	18343.	28454.	0.32727	0.3466	0.125553E-01
10.0909	0.1534553E+08	18494.	28454.	0.33636	0.3527	0.126304E-01
10.3636	0.1560976E+08	18631.	28454.	0.34545	0.3588	0.127067E-01

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10.6364	[illegible]	18755.	20454.	0.35455	0.3649	0.1276490E-01
10.9091	0.1601234E+00	18866.	20454.	0.36364	0.3709	0.1290621E-01
11.1818	0.1629390E+00	18964.	20454.	0.37273	0.3768	0.1297407E-01
11.4545	0.1662330E+00	19052.	20454.	0.38182	0.3828	0.1301935E-01
11.7273	0.1691200E+00	19129.	20454.	0.39091	0.3888	0.1307865E-01
12.0000	0.1717179E+00	19195.	20454.	0.40000	0.3947	0.1311775E-01
12.2727	0.1743314E+00	19252.	20454.	0.40909	0.4007	0.1325626E-01
12.5455	0.1769514E+00	19300.	20454.	0.41818	0.4067	0.1333466E-01
12.8182	0.1795222E+00	19340.	20454.	0.42727	0.4127	0.1341255E-01
13.0909	0.1821379E+00	19371.	20454.	0.43636	0.4187	0.1348995E-01
13.3636	0.1847679E+00	19394.	20454.	0.44545	0.4247	0.1356655E-01
13.6364	0.1874194E+00	19410.	20454.	0.45455	0.4308	0.1364235E-01
13.9091	0.1900730E+00	19416.	20454.	0.46364	0.4369	0.1371726E-01
14.1818	0.1927530E+00	19420.	20454.	0.47273	0.4431	0.1379125E-01
14.4545	0.1954530E+00	19415.	20454.	0.48182	0.4493	0.1386415E-01
14.7273	0.1981779E+00	19403.	20454.	0.49091	0.4556	0.1393595E-01
15.0000	0.2009927E+00	19385.	20454.	0.50000	0.4619	0.1400655E-01
15.2727	0.2037616E+00	19361.	20454.	0.50909	0.4683	0.1407595E-01
15.5455	0.2065694E+00	19332.	20454.	0.51818	0.4747	0.1414395E-01
15.8182	0.2093300E+00	19296.	20454.	0.52727	0.4812	0.1421075E-01
16.0909	0.2122900E+00	19256.	20454.	0.53636	0.4878	0.1427605E-01
16.3636	0.2151590E+00	19210.	20454.	0.54545	0.4944	0.1433995E-01
16.6364	0.2180300E+00	19159.	20454.	0.55455	0.5012	0.1440235E-01
16.9091	0.2220999E+00	19103.	20454.	0.56364	0.5080	0.1446325E-01
17.1818	0.2249000E+00	19042.	20454.	0.57273	0.5149	0.1452206E-01
17.4545	0.2270530E+00	18976.	20454.	0.58182	0.5219	0.1458045E-01
17.7273	0.2302140E+00	18905.	20454.	0.59091	0.5290	0.1463675E-01
18.0000	0.2332710E+00	18831.	20454.	0.60000	0.5362	0.1469135E-01
18.2727	0.2364430E+00	18751.	20454.	0.60909	0.5435	0.1474435E-01
18.5455	0.2396600E+00	18668.	20454.	0.61818	0.5509	0.1479565E-01
18.8182	0.2427330E+00	18580.	20454.	0.62727	0.5584	0.1484535E-01
19.0909	0.2462497E+00	18488.	20454.	0.63636	0.5661	0.1489335E-01
19.3636	0.2495014E+00	18392.	20454.	0.64545	0.5738	0.1493975E-01
19.6364	0.2535031E+00	18292.	20454.	0.65455	0.5816	0.1498435E-01
19.9091	0.2565601E+00	18188.	20454.	0.66364	0.5896	0.1502735E-01
20.1818	0.2590023E+00	18080.	20454.	0.67273	0.5977	0.1506865E-01
20.4545	0.2630030E+00	17968.	20454.	0.68182	0.6060	0.1510815E-01
20.7273	0.2667241E+00	17853.	20454.	0.69091	0.6143	0.1514605E-01
21.0000	0.2709300E+00	17734.	20454.	0.70000	0.6228	0.1518215E-01
21.2727	0.2746091E+00	17611.	20454.	0.70909	0.6314	0.1521655E-01
21.5455	0.2783000E+00	17485.	20454.	0.71818	0.6402	0.1524935E-01
21.8182	0.2822325E+00	17355.	20454.	0.72727	0.6491	0.1528035E-01
22.0909	0.2863272E+00	17222.	20454.	0.73636	0.6582	0.1530975E-01
22.3636	0.2903352E+00	17086.	20454.	0.74545	0.6674	0.1533745E-01
22.6364	0.2944670E+00	16946.	20454.	0.75455	0.6768	0.1536345E-01
22.9091	0.2985522E+00	16802.	20454.	0.76364	0.6863	0.1538785E-01
23.1818	0.3027842E+00	16656.	20454.	0.77273	0.6960	0.1541055E-01
23.4545	0.3070470E+00	16506.	20454.	0.78182	0.7058	0.1543155E-01
23.7273	0.3114030E+00	16353.	20454.	0.79091	0.7158	0.1545095E-01
24.0000	0.3158322E+00	16196.	20454.	0.80000	0.7260	0.1546875E-01
24.2727	0.3203370E+00	16037.	20454.	0.80909	0.7364	0.1548495E-01
24.5455	0.3249170E+00	15874.	20454.	0.81818	0.7469	0.1549955E-01
24.8182	0.3295772E+00	15708.	20454.	0.82727	0.7576	0.1551265E-01
25.0909	0.3342170E+00	15539.	20454.	0.83636	0.7685	0.1552415E-01
25.3636	0.3391400E+00	15368.	20454.	0.84545	0.7796	0.1553405E-01
25.6364	0.3440430E+00	15193.	20454.	0.85455	0.7909	0.1554245E-01

25.9091	0.349131E+00	15015.	28454.	0.86364	0.8023	0.155493E-01
26.1818	0.354160E+00	14834.	28454.	0.87273	0.8140	0.155547E-01
26.4545	0.359260E+00	14650.	28454.	0.88182	0.8258	0.155587E-01
26.7273	0.364515E+00	14463.	28454.	0.89091	0.8379	0.155611E-01
27.0000	0.369250E+00	14274.	28454.	0.90000	0.8502	0.155622E-01
27.2727	0.375280E+00	14081.	28454.	0.90909	0.8627	0.155618E-01
27.5455	0.38[illegible]210E+00	13886.	28454.	0.91818	0.8754	0.155601E-01
27.8182	0.386434E+00	13687.	28454.	0.92727	0.8883	0.155569E-01
28.0909	0.392152E+00	13486.	28454.	0.93636	0.9014	0.155525E-01
28.3636	0.397960E+00	13282.	28454.	0.94545	0.9148	0.155466E-01
28.6364	0.403884E+00	13075.	28454.	0.95455	0.9284	0.155395E-01
28.9091	0.409922E+00	12866.	28454.	0.96364	0.9422	0.155311E-01
29.1818	0.416[illegible]24E+00	12654.	28454.	0.97273	0.9563	0.155214E-01
29.4545	0.422250E+00	12438.	28454.	0.98182	0.9706	0.155105E-01
29.7273	0.428584E+00	12221.	28454.	0.99091	0.9852	0.154983E-01
30.0000	0.435[illegible]25E+00	12000.	28454.	1.00000	1.0000	0.154849E-01

ROTATING ANNULAR DISK STRESSED ON [illegible]
ORTHOTROPIC RATIO EQUALS 2.5, POISSONS RATIO EQUALS 0.5
CONSTANT HOOP STRESS CRITERION

DETERMINATION OF MODULUS VARIATION IN AN ANNULAR DISK

INPUT DATA
****

OUTER RADIUS = 30.0000 INNER RADIUS = 3.0000 RPM = 4000.00

RADIAL STRESS, OUTER RADIUS = 12000.00 RADIAL STRESS, INNER RADIUS = 0.00

ETHETA AT INNER RADIUS = 0.60000000E+07 POISSONS RATIO = 0.50000000 ORTHOTROPIC RATIO = 2.5000

MATERIAL DENSITY = 0.10000 NUMBER OF RADIAL POINTS = 100

OUTPUT DATA
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RADIUS ******	ETHETA ******	SIGMA-R *******	SIGMA-T *******	R/RO ****	E/EO ****	U-SUB-R *******
3.0000	0.600000E+07	0	28454.	0.10000	0.7633	0.142272E-01
3.2727	0.650730E+07	2334.	28454.	0.10909	0.8278	0.137236E-01
3.5455	0.692090E+07	4313.	28454.	0.11818	0.8803	0.134762E-01
3.8182	0.724470E+07	5964.	28454.	0.12727	0.9216	0.134194E-01
4.0909	0.749060E+07	7434.	28454.	0.13636	0.9528	0.135109E-01
4.3636	0.766530E+07	8697.	28454.	0.14545	0.9751	0.137227E-01
4.6364	0.777973E+07	9806.	28454.	0.15455	0.9897	0.140357E-01
4.9091	0.784220E+07	10784.	28454.	0.16364	0.9976	0.143366E-01
5.1818	0.786095E+07	11653.	28454.	0.17273	1.0000	0.147160E-01
5.4545	0.784310E+07	12429.	28454.	0.18182	0.9977	0.154669E-01
5.7273	0.779530E+07	13125.	28454.	0.19091	0.9917	0.160844E-01
6.0000	0.772365E+07	13750.	28454.	0.20000	0.9825	0.167648E-01
6.2727	0.763120E+07	14315.	28454.	0.20909	0.9708	0.175055E-01
6.5455	0.752400E+07	14827.	28454.	0.21818	0.9571	0.183044E-01
6.8182	0.740501E+07	15291.	28454.	0.22727	0.9420	0.191800E-01
7.0909	0.727700E+07	15713.	28454.	0.23636	0.9257	0.200712E-01
7.3636	0.714270E+07	16097.	28454.	0.24545	0.9086	0.210371E-01
7.6364	0.700420E+07	16447.	28454.	0.25455	0.8910	0.220570E-01
7.9091	0.686360E+07	16766.	28454.	0.26364	0.8731	0.231305E-01
8.1818	0.672181E+07	17058.	28454.	0.27273	0.8550	0.242570E-01
8.4545	0.657822E+07	17324.	28454.	0.28182	0.8369	0.254363E-01
8.7273	0.643740E+07	17567.	28454.	0.29091	0.8189	0.266679E-01
9.0000	0.629802E+07	17789.	28454.	0.30000	0.8012	0.279516E-01
9.2727	0.616100E+07	17991.	28454.	0.30909	0.7837	0.292869E-01
9.5455	0.602284E+07	18175.	28454.	0.31818	0.7667	0.306738E-01
9.8182	0.589594E+07	18343.	28454.	0.32727	0.7500	0.321112E-01
10.0909	0.576850E+07	18494.	28454.	0.33636	0.7338	0.335993E-01
10.3636	0.564492E+07	18631.	28454.	0.34545	0.7161	0.351373E-01

10.6364	0.552513E+07	18755.	28454.	0.35455	0.7029	0.367248E-01
10.9091	0.540930E+07	18866.	28454.	0.36364	0.6881	0.383611E-01
11.1818	0.529757E+07	18964.	28454.	0.37273	0.6739	0.400455E-01
11.4545	0.518988E+07	19052.	28454.	0.38182	0.6602	0.417772E-01
11.7273	0.508108E+07	19129.	28454.	0.39091	0.6470	0.435555E-01
12.0000	0.498845E+07	19195.	28454.	0.40000	0.6343	0.453794E-01
12.2727	0.486967E+07	19252.	28454.	0.40909	0.6221	0.472479E-01
12.5455	0.475877E+07	19300.	28454.	0.41818	0.6105	0.491600E-01
12.8182	0.471167E+07	19340.	28454.	0.42727	0.5993	0.511148E-01
13.0909	0.462827E+07	19371.	28454.	0.43636	0.5865	0.531104E-01
13.3636	0.454549E+07	19394.	28454.	0.44545	0.5782	0.551462E-01
13.6364	0.448824E+07	19410.	28454.	0.45455	0.5664	0.572207E-01
13.9091	0.439441E+07	19418.	28454.	0.46364	0.5590	0.593323E-01
14.1818	0.432399E+07	19420.	28454.	0.47273	0.5500	0.614797E-01
14.4545	0.425802E+07	19415.	28454.	0.48182	0.5415	0.636613E-01
14.7273	0.419240E+07	19403.	28454.	0.49091	0.5333	0.658754E-01
15.0000	0.413133E+07	19385.	28454.	0.50000	0.5256	0.681204E-01
15.2727	0.407314E+07	19361.	28454.	0.50909	0.5181	0.703947E-01
15.5455	0.401778E+07	19332.	28454.	0.51818	0.5111	0.726963E-01
15.8182	0.396510E+07	19296.	28454.	0.52727	0.5044	0.750235E-01
16.0909	0.391521E+07	19256.	28454.	0.53636	0.4981	0.773745E-01
16.3636	0.386782E+07	19210.	28454.	0.54545	0.4920	0.797472E-01
16.6364	0.382292E+07	19159.	28454.	0.55455	0.4863	0.821398E-01
16.9091	0.378142E+07	19103.	28454.	0.56364	0.4809	0.845503E-01
17.1818	0.374025E+07	19042.	28454.	0.57273	0.4758	0.869767E-01
17.4545	0.370235E+07	18976.	28454.	0.58182	0.4710	0.894169E-01
17.7273	0.366802E+07	18905.	28454.	0.59091	0.4664	0.918689E-01
18.0000	0.363302E+07	18831.	28454.	0.60000	0.4622	0.943306E-01
18.2727	0.360147E+07	18751.	28454.	0.60909	0.4581	0.968000E-01
18.5455	0.357194E+07	18666.	28454.	0.61818	0.4544	0.992749E-01
18.8182	0.354426E+07	18580.	28454.	0.62727	0.4509	0.101753E+00
19.0909	0.351853E+07	18488.	28454.	0.63636	0.4476	0.104233E+00
19.3636	0.349460E+07	18392.	28454.	0.64545	0.4446	0.106712E+00
19.6364	0.347245E+07	18292.	28454.	0.65455	0.4417	0.109188E+00
19.9091	0.345202E+07	18188.	28454.	0.66364	0.4391	0.111660E+00
20.1818	0.343320E+07	18080.	28454.	0.67273	0.4368	0.114124E+00
20.4545	0.341617E+07	17968.	28454.	0.68182	0.4346	0.116580E+00
20.7273	0.340027E+07	17853.	28454.	0.69091	0.4326	0.119024E+00
21.0000	0.338872E+07	17734.	28454.	0.70000	0.4308	0.121456E+00
21.2727	0.337431E+07	17611.	28454.	0.70909	0.4293	0.123873E+00
21.5455	0.336336E+07	17485.	28454.	0.71818	0.4279	0.126273E+00
21.8182	0.335291E+07	17355.	28454.	0.72727	0.4267	0.128654E+00
22.0909	0.334500E+07	17222.	28454.	0.73636	0.4256	0.131015E+00
22.3636	0.333924E+07	17086.	28454.	0.74545	0.4248	0.133353E+00
22.6364	0.333379E+07	16946.	28454.	0.75455	0.4241	0.135667E+00
22.9091	0.333060E+07	16802.	28454.	0.76364	0.4236	0.137956E+00
23.1818	0.332751E+07	16656.	28454.	0.77273	0.4233	0.140217E+00
23.4545	0.332625E+07	16506.	28454.	0.78182	0.4231	0.142449E+00
23.7273	0.332620E+07	16353.	28454.	0.79091	0.4231	0.144650E+00
24.0000	0.332750E+07	16196.	28454.	0.80000	0.4233	0.146819E+00
24.2727	0.333014E+07	16037.	28454.	0.80909	0.4236	0.148955E+00
24.5455	0.333374E+07	15874.	28454.	0.81818	0.4241	0.151055E+00
24.8182	0.333857E+07	15708.	28454.	0.82727	0.4248	0.153120E+00
25.0909	0.334322E+07	15539.	28454.	0.83636	0.4256	0.155146E+00
25.3636	0.335260E+07	15368.	28454.	0.84545	0.4285	0.157134E+00
25.6364	0.336133E+07	15193.	28454.	0.85455	0.4276	0.159082E+00

25.9091	0.331711E+07	15315.	28454.	0.86364	0.4289	0.160989E+00
26.1818	0.332210E+07	14834.	28454.	0.87273	0.4303	0.162853E+00
26.4545	0.339430E+07	14650.	28454.	0.88182	0.4318	0.164675E+00
26.7273	0.334077E+07	14463.	28454.	0.89091	0.4335	0.166452E+00
27.0000	0.342220E+07	14274.	28454.	0.90000	0.4354	0.168185E+00
27.2727	0.343797E+07	14081.	28454.	0.90909	0.4373	0.169872E+00
27.5455	0.345483E+07	13886.	28454.	0.91818	0.4395	0.171513E+00
27.8182	0.347264E+07	13687.	28454.	0.92727	0.4418	0.173107E+00
28.0909	0.349202E+07	13486.	28454.	0.93636	0.4442	0.174653E+00
28.3636	0.351230E+07	13282.	28454.	0.94545	0.4468	0.176151E+00
28.6364	0.353367E+07	13075.	28454.	0.95455	0.4495	0.177600E+00
28.9091	0.355654E+07	12866.	28454.	0.96364	0.4524	0.179001E+00
29.1818	0.358036E+07	12654.	28454.	0.97273	0.4555	0.180352E+00
29.4545	0.360539E+07	12438.	28454.	0.98182	0.4586	0.181653E+00
29.7273	0.363157E+07	12221.	28454.	0.99091	0.4620	0.182904E+00
30.0000	0.365857E+07	12000.	28454.	1.00000	0.4655	0.184105E+00

ROTATING ANNULAR DISK STRESSED ON O.D.
ORTHOTROPIC RATIO EQUALS 5.0, POISSONS RATIO EQUALS 0.5
CONSTANT HOOP STRESS CRITERION

DETERMINATION OF MODULUS VARIATION IN AN ANNULAR DISK

INPUT DATA
****

OUTER RADIUS = 30.0000 INNER RADIUS = 3.0000 RPM = 4000.00

RADIAL STRESS, OUTER RADIUS = 12000.00 RADIAL STRESS, INNER RADIUS = 0.00

ETHETA AT INNER RADIUS = 0.60000000E+07 POISSONS RATIO = 0.50000000 ORTHOTROPIC RATIO = 5.0000

MATERIAL DENSITY = 0.10000 NUMBER OF RADIAL POINTS = 100

OUTPUT DATA
****

RADIUS	ETHETA	SIGMA-R	SIGMA-T	R/RO	E/EO	U-SUB-R
******	******	*******	*******	****	****	*******
3.0000	0.600000E+07	0	28454.	0.10000	0.8883	0.142272E-01
3.2727	0.645133E+07	2334.	28454.	0.10909	0.9551	0.138428E-01
3.5455	0.669355E+07	4303.	28454.	0.11818	0.9910	0.139323E-01
3.8182	0.675433E+07	5984.	28454.	0.12727	1.0000	0.143939E-01
4.0909	0.666781E+07	7434.	28454.	0.13636	0.9872	0.151771E-01
4.3636	0.646859E+07	8697.	28454.	0.14545	0.9577	0.162606E-01
4.6364	0.618952E+07	9806.	28454.	0.15455	0.9164	0.176418E-01
4.9091	0.585684E+07	10784.	28454.	0.16364	0.8671	0.193305E-01
5.1818	0.549330E+07	11653.	28454.	0.17273	0.8133	0.213457E-01
5.4545	0.511547E+07	12429.	28454.	0.18182	0.7574	0.237141E-01
5.7273	0.473702E+07	13125.	28454.	0.19091	0.7013	0.264686E-01
6.0000	0.436766E+07	13750.	28454.	0.20000	0.6466	0.296482E-01
6.2727	0.401197E+07	14315.	28454.	0.20909	0.5940	0.332974E-01
6.5455	0.367594E+07	14827.	28454.	0.21818	0.5442	0.374662E-01
6.8182	0.336130E+07	15291.	28454.	0.22727	0.4977	0.422099E-01
7.0909	0.306913E+07	15713.	28454.	0.23636	0.4544	0.475896E-01
7.3636	0.279963E+07	16097.	28454.	0.24545	0.4145	0.536716E-01
7.6364	0.255237E+07	16447.	28454.	0.25455	0.3779	0.605283E-01
7.9091	0.232630E+07	16766.	28454.	0.26364	0.3444	0.682375E-01
8.1818	0.212040E+07	17058.	28454.	0.27273	0.3139	0.768829E-01
8.4545	0.193330E+07	17324.	28454.	0.28182	0.2862	0.865543E-01
8.7273	0.176351E+07	17567.	28454.	0.29091	0.2611	0.973473E-01
9.0000	0.160960E+07	17789.	28454.	0.30000	0.2383	0.109363E+00
9.2727	0.147043E+07	17991.	28454.	0.30909	0.2177	0.122710E+00
9.5455	0.134445E+07	18175.	28454.	0.31818	0.1991	0.137501E+00
9.8182	0.123054E+07	18343.	28454.	0.32727	0.1822	0.153856E+00
10.0909	0.112754E+07	18494.	28454.	0.33636	0.1669	0.171898E+00
10.3636	0.103435E+07	18631.	28454.	0.34545	0.1531	0.191760E+00

10.6364	0.23[illegible]63E+00	18755.	28454.	0.35455	0.1407	0.213575E+00
10.9091	0.813777E+00	18868.	28454.	0.36364	0.1294	0.237485E+00
11.1818	0.6[illegible]491E+00	18964.	28454.	0.37273	0.1191	0.263635E+00
11.4545	0.74[illegible]E+00	19052.	28454.	0.38182	0.1099	0.292174E+00
11.7273	0.605311E+00	19129.	28454.	0.39091	0.1015	0.323255E+00
12.0000	0.633722E+00	19195.	28454.	0.40000	0.0938	0.357034E+00
12.2727	0.586975E+00	19252.	28454.	0.40909	0.0869	0.393671E+00
12.5455	0.544410E+00	19300.	28454.	0.41818	0.0806	0.433327E+00
12.8182	0.5[illegible]674E+00	19340.	28454.	0.42727	0.0749	0.476164E+00
13.0909	0.470303E+00	19371.	28454.	0.43636	0.0696	0.522347E+00
13.3636	0.432200E+00	19394.	28454.	0.44545	0.0649	0.572038E+00
13.6364	0.4[illegible]817E+00	19410.	28454.	0.45455	0.0605	0.625400E+00
13.9091	0.381966E+00	19418.	28454.	0.46364	0.0566	0.682597E+00
14.1818	0.357404E+00	19420.	28454.	0.47273	0.0529	0.743786E+00
14.4545	0.334967E+00	19415.	28454.	0.48182	0.0496	0.809124E+00
14.7273	0.314283E+00	19403.	28454.	0.49091	0.0465	0.878764E+00
15.0000	0.299538E+00	19385.	28454.	0.50000	0.0437	0.952853E+00
15.2727	0.277962E+00	19361.	28454.	0.50909	0.0412	0.103153E+01
15.5455	0.261967E+00	19332.	28454.	0.51818	0.0388	0.111494E+01
15.8182	0.247241E+00	19296.	28454.	0.52727	0.0366	0.120320E+01
16.0909	0.233089E+00	19256.	28454.	0.53636	0.0346	0.129644E+01
16.3636	0.221147E+00	19210.	28454.	0.54545	0.0327	0.139476E+01
16.6364	0.209584E+00	19159.	28454.	0.55455	0.0310	0.149827E+01
16.9091	0.198295E+00	19103.	28454.	0.56364	0.0294	0.160706E+01
17.1818	0.18[illegible]E+00	19042.	28454.	0.57273	0.0280	0.172120E+01
17.4545	0.179643E+00	18976.	28454.	0.58182	0.0266	0.184076E+01
17.7273	0.171354E+00	18905.	28454.	0.59091	0.0254	0.196581E+01
18.0000	0.163473E+00	18831.	28454.	0.60000	0.0242	0.209637E+01
18.2727	0.156109E+00	18751.	28454.	0.60909	0.0231	0.223247E+01
18.5455	0.149200E+00	18668.	28454.	0.61818	0.0221	0.237414E+01
18.8182	0.143[illegible]33E+00	18580.	28454.	0.62727	0.0212	0.252135E+01
19.0909	0.137147E+00	18488.	28454.	0.63636	0.0203	0.267411E+01
19.3636	0.131222E+00	18392.	28454.	0.64545	0.0195	0.283237E+01
19.6364	0.126340E+00	18292.	28454.	0.65455	0.0187	0.299609E+01
19.9091	0.121770E+00	18188.	28454.	0.66364	0.0180	0.316520E+01
20.1818	0.117325E+00	18080.	28454.	0.67273	0.0174	0.333963E+01
20.4545	0.113163E+00	17968.	28454.	0.68182	0.0168	0.351927E+01
20.7273	0.109277E+00	17853.	28454.	0.69091	0.0162	0.370401E+01
21.0000	0.105841E+00	17734.	28454.	0.70000	0.0156	0.389374E+01
21.2727	0.102237E+00	17611.	28454.	0.70909	0.0151	0.408829E+01
21.5455	0.990534E-01	17485.	28454.	0.71818	0.0147	0.428753E+01
21.8182	0.960743E-01	17355.	28454.	0.72727	0.0142	0.449126E+01
22.0909	0.932813E-01	17222.	28454.	0.73636	0.0138	0.469932E+01
22.3636	0.906640E-01	17086.	28454.	0.74545	0.0134	0.491149E+01
22.6364	0.882122E-01	16946.	28454.	0.75455	0.0131	0.512756E+01
22.9091	0.859130E-01	16802.	28454.	0.76364	0.0127	0.534730E+01
23.1818	0.837581E-01	16656.	28454.	0.77273	0.0124	0.557049E+01
23.4545	0.817375E-01	16506.	28454.	0.78182	0.0121	0.579685E+01
23.7273	0.798431E-01	16353.	28454.	0.79091	0.0118	0.602615E+01
24.0000	0.780875E-01	16196.	28454.	0.80000	0.0116	0.625810E+01
24.2727	0.764030E-01	16037.	28454.	0.80909	0.0113	0.649242E+01
24.5455	0.748435E-01	15874.	28454.	0.81818	0.0111	0.672883E+01
24.8182	0.733830E-01	15708.	28454.	0.82727	0.0109	0.696704E+01
25.0909	0.720150E-01	15539.	28454.	0.83636	0.0107	0.720674E+01
25.3636	0.707365E-01	15368.	28454.	0.84545	0.0105	0.744764E+01
25.6364	0.695400E-01	15193.	28454.	0.85455	0.0103	0.768941E+01

25.9091	0.604237E+05	15215.	20454.	0.88364	0.0101	0.793174E+01
26.1818	0.573217E+05	14834.	20454.	0.87273	0.0100	0.817432E+01
26.4545	0.564105E+05	14650.	20454.	0.88182	0.0098	0.841084E+01
26.7273	0.555[illegible]75E+05	14463.	20454.	0.89091	0.0097	0.865097[illegible]+01
27.0000	0.548888E+05	14279.	20454.	0.90000	0.0096	0.890040E+01
27.2727	0.538911E+05	14081.	20454.	0.90909	0.0095	0.914081E+01
27.5455	0.531723E+05	13886.	20454.	0.91818	0.0094	0.937988E+01
27.8182	0.625[illegible]93E+05	13687.	20454.	0.92727	0.0093	0.961731E+01
28.0909	0.619[illegible]05E+05	13486.	20454.	0.93636	0.0092	0.985279E+01
28.3636	0.513430E+05	13282.	20454.	0.94545	0.0091	0.100860E+02
28.6364	0.50[illegible]635E+05	13075.	20454.	0.95455	0.0090	0.10[illegible]167E+02
28.9091	0.503747E+05	12866.	20454.	0.96364	0.0089	0.105[illegible]45E+02
29.1818	0.599613E+05	12654.	20454.	0.97273	0.0089	0.107692E+02
29.4545	0.595902E+05	12438.	20454.	0.98182	0.0088	0.109905E+02
29.7273	0.592630E+05	12221.	20454.	0.99091	0.0088	0.112082E+02
30.0000	0.589773E+05	12000.	20454.	1.00000	0.0087	0.114219E+02